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Aspects of the Stueckelberg extension

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Abstract: A detailed analysis of a Stueckelberg extension of the electro-weak gauge group with an extra abelian $\mathrm{U}(1)_X$ factor is presented for the Standard Model as well as for the MSSM. The extra gauge boson gets massive through a Stueckelberg type coupling to a pseudo-scalar, instead of a Higgs effect. This new massive neutral gauge boson Z' has vector and axial vector couplings uniquely different from those of conventional extra abelian gauge bosons, such as appear e.g. in GUT models. The extended MSSM furthermore contains two extra neutralinos and one extra neutral CP-even scalar, the latter with a mass larger than that of the Z'. One interesting scenario that emerges is an LSP that is dominantly composed out of the new neutralinos, leading to a possible new superweak candidate for dark matter. We investigate signatures of the Stueckelberg extension at a linear collider and discuss techniques for the detection of the expected sharp Z' resonance. It turns out that the substantially modified forward-backward asymmetry around the Z' pole provides an important signal. Furthermore, we also elaborate on generalizations of the minimal Stueckelberg extension to an arbitrary number of extra U(1) gauge factors.

Keywords: Supersymmetry Phenomenology, Supersymmetric Standard Model, Beyond Standard Model.

Contents

1. Introduction

The Stueckelberg lagrangian [1] is a gauge invariant kinetic term for a massive abelian vector field, that utilizes a non-linear representation of the gauge transformation. The mass term is made gauge invariant by coupling a massless gauge boson to a real pseudo-scalar, which then transforms non-linearly, and in unitary gauge is absorbed as the longitudinal mode of the massive vector. As we shall point out below, gauge boson masses through Stueckelberg couplings are ubiquitous in compactifications of higher-dimensional string theory, supergravity, or even pure gauge theory. From a model building perspective, the relevance of the Stueckelberg mechanism lies in the fact that it provides an opportunity alternative to the Higgs mechanism [2] to achieve gauge symmetry breaking without spoiling renormalizability [3]. Since the minimal version of the Stueckelberg mechanism only needs a single real scalar, which is absorbed by the gauge boson with no other degrees of freedom left, it is already clear that the Stueckelberg and the Higgs mechanism are physically distinct. The main purpose of this paper is to discuss the most simple extensions of the electro-weak sector of the Standard Model (SM) [4], and its supersymmetric generalizations (SSM or MSSM). The work presented here is a more detailed exposition and extension of of two previous publications, where the Stueckelberg extension was first achieved [5–7]. In particular, an analysis of the possibility of observation of Stueckelberg phenomena at linear colliders is also given.

1.1 The Stueckelberg lagrangian

The prototype Stueckelberg lagrangian couples one abelian vector boson A_μ to one pseudo-scalar σ in the following way,[1]

$$\mathcal{L} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{2}(mA_\mu + \partial_\mu\sigma)(mA^\mu + \partial^\mu\sigma)\,. \tag{1.1}$$

It is gauge invariant if σ transforms together with A_μ according to

$$\delta A_\mu = \partial_\mu\epsilon\,, \qquad \delta\sigma = -m\epsilon\,. \tag{1.2}$$

Fixing the gauge by adding

$$\mathcal{L}_{\text{gf}} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu + \xi m\sigma\right)^2\,, \tag{1.3}$$

the total lagrangian reads

$$\mathcal{L} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{gf}} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 - \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \xi\frac{m^2}{2}\sigma^2 \tag{1.4}$$

[1] As is well known, the Stueckelberg mechanism can actually be recovered in a rather singular limit of the Higgs mechanism [3], and it is useful to keep the comparison in mind as we discuss the models based on the Stueckelberg mechanism. This similarity, however, is not so easily realized in the case of the supersymmetric Stueckelberg extension of the MSSM.

where the two fields have been decoupled, and renormalizability and unitarity are manifest. To add interactions with fermions, one may couple the vector field to a conserved current, adding the interaction

$$\mathcal{L}_{\rm int} = g A_\mu J^\mu \tag{1.5}$$

with $\partial_\mu J^\mu = 0$.

Let us mention here that regarding the extension of this mechanism to non-abelian gauge theories, according to [8], a non-abelian extension of the Stueckelberg lagrangian leads to violation of unitarity already at the tree-level, because the longitudinal components of the vector fields cannot be decoupled from the physical Hilbert space. The renormalizability of the theory is then spoiled as well. Therefore, the Higgs of the SM cannot be replaced by a Stueckelberg type of symmetry breaking. Instead we will consider extensions of the SM or the MSSM which involve extra U(1) gauge factors beyond the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge symmetry of the SM, which will then be assumed to couple to pseudo-scalars in the way of Stueckelberg.

1.2 Stueckelberg in string theory and compactification

One immediate way to see that Stueckelberg couplings appear in dimensional reduction of supergravity from higher dimensions, and in particular string theory, is to consider the reduction of the ten-dimensional $N = 1$ supergravity coupled to supersymmetric Yang-Mills gauge fields [9], in the presence of internal gauge fluxes. The ten-dimensional kinetic term for the anti-symmetric 2-tensor B_{IJ} involves a coupling to the Yang-Mills Chern-Simons form, schematically $\partial_{[I} B_{JK]} + A_{[I} F_{JK]} + \frac{2}{3} A_{[I} A_J A_{K]}$, in proper units. Dimensional reduction with a vacuum expectation value for the internal gauge field strength, $\langle F_{ij} \rangle \neq 0$, leads to

$$\partial_\mu B_{ij} + A_\mu F_{ij} \;\sim\; \partial_\mu \sigma + m A_\mu \,, \tag{1.6}$$

after identifying the internal components B_{ij} with the scalar σ and the value of the gauge field strength with the mass parameter m. Thus, A_μ and σ have a Stueckelberg coupling of the form $A_\mu \partial^\mu \sigma$. These couplings play an important role in the Green-Schwarz anomaly cancellation mechanism. In a four-dimensional theory abelian gauge symmetries can have a triangle ABJ anomaly, if $\mathrm{tr}\, Q \neq 0$ or $\mathrm{tr}\, Q^3 \neq 0$. In a consistent string compactification, this ABJ anomaly is cancelled by Green-Schwarz type contributions involving the two terms $m A^\mu \partial_\mu \sigma + c\, \sigma F_{\mu\nu} \tilde{F}^{\mu\nu}$ in the lagrangian and the anomalous 3-point function is proportional to the product of the two couplings, $m \cdot c$, while the mass parameter in the Stueckelberg coupling is only m. Therefore, any anomalous U(1) will always get massive through the Stueckelberg mechanism, since $m \cdot c \neq 0$, but a non-anomalous U(1) can do so as well, if $m \neq 0$, $c = 0$. Since we do not want to deal with anomalous gauge symmetries here, we shall always assume that $m \neq 0$, $c = 0$. The mass scale that determines m within models that derive from string theory can, at leading order, also be derived from dimensional reduction. It turns out to be proportional to the string or compactification scale in many cases [10], but can in principle also be independent [11].

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The fact that an abelian gauge symmetry, anomalous or non-anomalous, may decouple from the low energy theory via Stueckelberg couplings was actually of great importance in the construction of D-brane models with gauge group and spectrum close to that of the SM [12]. Roughly speaking, these D-brane constructions start with a number of unitary gauge group factors U(N), which are then usually broken to their subgroups SU(N) via Stueckelberg couplings,

$$\mathrm{U}(3) \times \mathrm{U}(2) \times \mathrm{U}(1)^2 \quad \stackrel{\text{Stueckelberg}}{\longrightarrow} \quad \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \,. \tag{1.7}$$

The mass matrix for the abelian gauge bosons is then block-diagonal, and only the SM survives. In order to ensure this pattern, one has to impose a condition on the Stueckelberg mass parameters, namely that the hyper charge gauge boson does not couple to any axionic scalar and remains massless [12]. In the language of these D-brane models, we will here relax this extra condition, and explore the consequences of letting the hyper charge gauge boson mix with other abelian gauge factors beyond the SM gauge group, which seems a very natural extension of the SM in this frame work.

In a much simpler framework, in the dimensional compactification of abelian gauge theory on a circle, one can also demonstrate that the higher Kaluza-Klein excitations of the vector field gain their mass through a Stueckelberg mechanism. For this purpose we consider a five-dimensional abelian gauge field A_I, using coordinates $x_I = (x_\mu, y)$. The gauge kinetic energy including a gauge fixing term is

$$\mathcal{L}_{5d} = -\frac{1}{4}\mathcal{F}_{IJ}(z)\mathcal{F}^{IJ} - \frac{1}{2\xi}(\partial_I A^I)^2 \,. \tag{1.8}$$

We compactify the fifth dimension on a half circle $\mathcal{S}^1$ of radius R and expand the five-dimensional gauge field $A_I(x_I) = (A_\mu(x_I), \sigma(x_I))$ in harmonics on the compactified dimension,

$$A_\mu(x_I) = \sum_{n=0}^{\infty} A_\mu^{(n)}(x_\mu)\xi_n(y) \,, \qquad \sigma(x_I) = \sum_{n=0}^{\infty} \sigma^{(n)}(x_\mu)\eta_n(y) \,, \tag{1.9}$$

where $\xi_n(y)$ and $\eta_n(y)$ are harmonic functions on the interval $(0, 2\pi R)$ with appropriate periodicity conditions. The effective lagrangian in four dimensions is obtained by integration over the fifth dimension,

$$\begin{aligned}\mathcal{L}_{4d} = \sum_{n=0}^{\infty} \Bigg[& -\frac{1}{4}\mathcal{F}_{\mu\nu}^{(n)}\mathcal{F}^{\mu\nu(n)} - \frac{1}{2}n^2(MA_\mu^{(n)} + n\partial_\mu\sigma^{(n)})^2 - \\ & - \frac{1}{2\xi}\Big[(\partial_\mu A^{(n)\mu})^2 + 2nM\partial_\mu A^{(n)\mu}\sigma^{(n)} + M^2(\sigma^{(n)})^2\Big]\Bigg] \,,\end{aligned} \tag{1.10}$$

where $M = 1/R$ is the inverse of the compactification radius. The Stueckelberg mechanism is now manifest in the first line of eq. (1.10). Choosing the gauge $\xi = 1$ one finds that the bilinear terms involving $A_\mu^{(n)}$ and $\sigma^{(n)}$ form a total divergence which can be discarded, and the scalar fields $\sigma^{(n)}$ decouple from the vector fields. One is thus left with one massless vector field and an infinite tower of massive vector fields all of which gain masses by the Stueckelberg mechanism. There is no Higgs phenomenon involved in the generation of their masses.

1.3 Overview and summary

The rest of the paper is devoted to further development of the Stueckelberg extension of the SM and of the MSSM, and applications to a number of phenomena which have the possibility of being tested in current and future experiment. The outline is as follows: In section 2 we give a detailed discussion of the extension of the SM electro-weak gauge group $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ to $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\times\mathrm{U}(1)_X$. We show that the Stueckelberg extension allows one to retain a massless mode which is identified with the photon, while the remaining two vector bosons become massive and correspond to the gauge bosons Z and Z$'$. Several useful results relating the mass parameters and the mixing angles are deduced and general formulae for the neutral current couplings to fermions are deduced. In section 3 we give a full analysis of the extension of MSSM to include a Stueckelberg $\mathrm{U}(1)_X$ gauge group, where in addition to a gauge vector multiplet for the $\mathrm{U}(1)_X$ one also has a chiral multiplet that involves the Stueckelberg pseudo-scalar. The Stueckelberg extension here reproduces the vector boson sector of the Stueckelberg extension of the SM and in addition contains new states and interactions including an additional spin zero state, an extra neutral gaugino and an extra neutral chiral fermion. In this section we also discuss the implications of including Fayet-Illiopoulos D-terms in the analysis.

In section 4 we discuss the implications and predictions of the Stueckelberg extensions. We work out in detail the deviations from the SM couplings in the neutral current sector and estimate the size of the parameters in the mixing of the Stueckelberg sector with the SM. Consistency with current data translates into bounds on these parameters. However, refined experiments should be able to discern deviations from the SM, such as the presence of a sharp Z$'$ resonance. A careful scanning of data will be needed to discern such a resonance. An explicit analysis of the modifications of the Z boson couplings and of the couplings of the Z$'$ boson to SM fermions shows that the Z$'$ has decay signatures which are very distinct from the Z, and the observation of such signatures should uniquely identify the Z$'$ boson. In this section we also discuss the mixing of the CP-even spin zero state from the Stueckelberg chiral multiplet with the two CP-even Higgs of the SM producing a 3×3 CP-even Higgs mass matrix. In the neutralino sector there are now two more neutral states arising from the Stueckelberg sector, which mix with the four neutralino states from the MSSM producing a 6×6 neutralino mass matrix. There exists a region of the parameter space where one of the Stueckelberg fermions is the LSP. This would have a drastic influence on collider signals for supersymmetry. Similarly, the dark matter relic abundance will be affected. Additional topics discussed in section 4 include the decay of Z$'$ into the hidden sector fermions which tend to give it a significantly larger decay width than what is allowed by the decays into the visible sector, and the modification of the correction to $g_\mu-2$ by inclusion of the Z$'$ boson exchange.

In section 5 a detailed investigation for testing the Stueckelberg scenario at a linear collider is performed. We analyze the e^+e^- cross section into leptons and quarks and also the forward-backward asymmetry A_{fb}. It is shown that in the vicinity of the Z$'$ resonance it deviates significantly from the SM prediction and hence will be a good indicator for discerning such a resonance. In section 6 we discuss briefly the technique that may be

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used for the detection of an expected sharp resonance for the Z′. In section 7 we include a generalization of the minimal Stueckelberg extension with just one extra U(1) to an arbitrary number of extra abelian factors. Section 8 is devoted to conclusions.

2. Stueckelberg extension of the standard model

We now turn to the main subject of this paper, the minimal extensions of the SM and the MSSM which involve Stueckelberg type couplings, and their experimental signatures. We start naturally with the SM, then discuss the supersymmetrized version for the MSSM, and afterward discuss the observable consequences. In any case, since the Stueckelberg is only compatible with abelian gauge symmetries, the minimal model that carries non-trivial structure is obtained by adding an abelian gauge group factor $U(1)_X$ to the SM gauge group, extending it to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$.[2] Then, all the abelian factors, i.e. hyper charge and $U(1)_X$, can couple to a real pseudo-scalar σ in the way of the Stueckelberg mechanism.[3] We call this model the StSM (or StMSSM for the supersymmetric version). In greater generality, one can of course add any number of abelian factors to the SM, and have all the abelian gauge bosons couple to any number of pseudo-scalars. In many string theoretic models based on D-branes and orientifolds, there is indeed a number of such gauge factors and scalars present, the maximum multiplicity being restricted by topological properties of the compactification space. We will come back to this option in section 7.

To start with the StSM [6], let A^a_μ, $a = 1, 2, 3$, be the vector fields in the adjoint of $SU(2)_L$, with field strength $F^a_{\mu\nu}$, B_μ the hyper charge vector with field strength $B_{\mu\nu}$, and Φ be the Higgs doublet.[4] Then the relevant part of the SM lagrangian is given by

$$\mathcal{L}_{\rm SM} = -\frac{1}{4}{\rm tr}\, F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + g_2 A^a_\mu J^{a\mu}_2 + g_Y B_\mu J^\mu_Y - D_\mu\Phi^\dagger D^\mu\Phi - V(\Phi^\dagger\Phi)\,, \quad (2.1)$$

where $D_\mu\Phi$ is the gauge covariant derivative. For the minimal Stueckelberg extension of this lagrangian, we add the degrees of freedom of one more abelian vector field C_μ for the $U(1)_X$, with field strength $C_{\mu\nu}$, and one pseudo-scalar σ. For the scalar field σ we assume that it will have Stueckelberg couplings to all the abelian gauge bosons, B_μ and C_μ. For the Higgs scalar Φ we assume that it is neutral under the $U(1)_X$, which just means that C_μ does not appear in $D_\mu\Phi$. This is an assumption somehow "orthogonal" to the starting point of most models with so-called $U(1)'$ gauge symmetries beyond the SM [15]. There, the gauge symmetry of the extra factor is broken by an extended Higgs effect. In our model, all non-trivial modification of the SM results from the Stueckelberg coupling and is mediated by the axion σ. Thus, the lagrangian of eq. (2.1) is extended to the StSM by

$$\mathcal{L}_{\rm StSM} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm St} \quad (2.2)$$

[2] In principle, one could also just consider a Stueckelberg coupling for only the hyper charge gauge boson, but this would ultimately give a non-vanishing mass to the photon, which is unacceptable.

[3] We frequently call this scalar an axionic scalar because of its pseudo-scalar nature, which does not imply that it couples to QCD gauge fields in the way of the usual QCD axion.

[4] The color $SU(3)_C$ factor of the gauge group will be irrelevant for most of what we have to say.

with

$$\mathcal{L}_{\rm St} = -\frac{1}{4}C_{\mu\nu}C^{\mu\nu} + g_X C_\mu J_X^\mu - \frac{1}{2}(\partial_\mu\sigma + M_1 C_\mu + M_2 B_\mu)^2 \,. \tag{2.3}$$

Up to this point, we have not specified the charges of the SM fermions, which in principle could carry charges under $\mathrm{U}(1)_X$. Later, we will, however, abandon this possibility. Furthermore, there may also be a sector that is hidden with respect to the SM gauge symmetries, i.e. neutral with respect to $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, but charged under $\mathrm{U}(1)_X$, and thus enters J_X^μ. In such a case, the Stueckelberg coupling would be the only way to communicate to the hidden sector. The extended non-linear gauge invariance now reads

$$\delta_Y B_\mu = \partial_\mu \lambda_Y \,, \qquad \delta_Y \sigma = -M_2 \lambda_Y \,, \tag{2.4}$$

for the hyper charge, and

$$\delta_X C_\mu = \partial_\mu \lambda_X \,, \qquad \delta_X \sigma = -M_1 \lambda_X \,. \tag{2.5}$$

for $\mathrm{U}(1)_X$. To decouple the two abelian gauge bosons from σ, one has to add a similar gauge fixing term as in the previous section with only one vector field. Furthermore, one has to add the standard gauge fixing terms for the charged gauge bosons to decouple from the Higgs.

2.1 Masses for the neutral vector bosons

After spontaneous electro-weak symmetry breaking the mass terms, with mass-squared matrix $M_{ab}^{[1]2}$ (upper index [1] for spin 1) for the neutral vector bosons $V_{\mu a} = (C_\mu, B_\mu, A_\mu^3)_a$, take the form

$$-\frac{1}{2}\sum_{a,b=1}^{3} V_{\mu a} M_{ab}^{[1]2} V_b^\mu \,, \tag{2.6}$$

where

$$M_{ab}^{[1]2} = \begin{bmatrix} M_1^2 & M_1 M_2 & 0 \\ M_1 M_2 & M_2^2 + \frac{1}{4}g_Y^2 v^2 & -\frac{1}{4}g_Y g_2 v^2 \\ 0 & -\frac{1}{4}g_Y g_2 v^2 & \frac{1}{4}g_2^2 v^2 \end{bmatrix} \tag{2.7}$$

where g_2 and g_Y are the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ gauge coupling constants, and are normalized so that $M_W^2 = g_2^2 v^2/4$. From $\det(M_{ab}^{[1]2}) = 0$ it is easily seen that one eigenvalue is zero, whose eigenvector we identify with the photon A_μ^γ,[5] and the remaining two eigenvalues are the roots

$$\begin{aligned} M_\pm^2 = \frac{1}{2}\Bigg[& M_1^2 + M_2^2 + \frac{1}{4}g_Y^2 v^2 + \frac{1}{4}g_2^2 v^2 \pm \\ & \pm \Bigg[\left(M_1^2 + M_2^2 + \frac{1}{4}g_Y^2 v^2 + \frac{1}{4}g_2^2 v^2\right)^2 - \left(M_1^2 (g_Y^2 + g_2^2) v^2 + g_2^2 M_2^2 v^2\right)\Bigg]^{\frac{1}{2}}\Bigg] \,. \end{aligned} \tag{2.8}$$

[5] Note that we succeeded in obtaining a massless photon, while previous attempts to obtain a Stueckelberg extension of the SM failed in this respect [13]. The basic reason for the difficulty in keeping the photon massless arose because there was an extra axion field which allowed the photon to generate a tiny mass. In our analysis we have an extra axion field σ and two gauge bosons, B_μ and C_μ. Thus, after absorption of the axion we are indeed left with a strictly massless photon field.

Obviously,

$$M_+^2 = M_1^2 + M_2^2 + \mathcal{O}(v^2)\,, \qquad M_-^2 = \mathcal{O}(v^2)\,. \tag{2.9}$$

We, therefore, identify the mass eigenstate with mass squared M_-^2 with the Z-boson, and call the mass eigenstate with eigenvalue M_+^2 the Z′-boson. The diagonal matrix of eigenvalues $E^{[1]}$ and the three eigenstates $E_\mu^{[1]}$ are denoted

$$E^{[1]} = \mathrm{diag}(M_{\mathrm{Z}'}^2, M_{\mathrm{Z}}^2, 0) = \mathrm{diag}(M_+^2, M_-^2, 0)\,, \qquad E_\mu^{[1]} = (\mathrm{Z}'_\mu, \mathrm{Z}_\mu, A_\mu^\gamma)^T\,. \tag{2.10}$$

Thus, we have

$$V_{\mu a} = \sum_{b=1}^{3} \mathcal{O}_{ab}^{[1]} E_{\mu b}^{[1]}\,, \qquad \sum_{b,c=1}^{3} \mathcal{O}_{ba}^{[1]} M_{bc}^{[1]2} \mathcal{O}_{cd}^{[1]} = E_{ad}^{[1]}\,, \tag{2.11}$$

for some orthogonal transformation matrix $\mathcal{O}_{ab}^{[1]}$. One can actually solve for it explicitly. We use the parametrization

$$\mathcal{O}^{[1]} = \begin{bmatrix} \cos\psi\cos\phi - \sin\theta\sin\phi\sin\psi & -\sin\psi\cos\phi - \sin\theta\sin\phi\cos\psi & -\cos\theta\sin\phi \\ \cos\psi\sin\phi + \sin\theta\cos\phi\sin\psi & -\sin\psi\sin\phi + \sin\theta\cos\phi\cos\psi & \cos\theta\cos\phi \\ -\cos\theta\sin\psi & -\cos\theta\cos\psi & \sin\theta \end{bmatrix}$$

Inverting the relation, one finds immediately

$$\tan(\phi) = \frac{M_2}{M_1} = \delta\,, \qquad \tan(\theta) = \frac{g_Y}{g_2}\cos(\phi) = \tan(\theta_W)\cos(\phi)\,. \tag{2.12}$$

Expressing $\tan(\psi)$ is a bit more tedious, and we use

$$\tan(\psi) = \frac{\tan(\theta)\tan(\phi) M_{\mathrm{W}}^2}{\cos(\theta)(M_{\mathrm{Z}'}^2 - M_{\mathrm{W}}^2(1+\tan^2(\theta)))}\,, \tag{2.13}$$

where $M_{\mathrm{W}} = g_2 v/2,\ \tan(\theta_W) = g_Y/g_2$. One can define the two independent parameters to describe the StSM extension,

$$\delta = \frac{M_2}{M_1}\,, \qquad M^2 = M_1^2 + M_2^2\,. \tag{2.14}$$

Effectively, M is the overall mass scale of the new physics, and δ the parameter that measures the strength of its coupling to the SM. In the limit $\delta \to 0$, where the SM and a decoupled abelian vector boson C_μ with mass M are recovered, one has

$$\tan(\phi)\,, \quad \tan(\psi) \;\longrightarrow\; 0\,, \qquad \tan(\theta) \;\longrightarrow\; \tan(\theta_{\mathrm{W}})\,, \tag{2.15}$$

i.e. θ becomes the weak angle, and the other angles vanish. In this limit the mass M_- takes the standard expression for the mass of the Z-boson,

$$M_{\mathrm{Z}'}^2 \;\longrightarrow\; M^2\,, \qquad M_{\mathrm{Z}}^2 \;\longrightarrow\; \frac{1}{4}v^2(g_2^2 + g_Y^2)\,, \tag{2.16}$$

the mass squared matrix being block-diagonal. Remarkably, in the limit $v/M \to 0$, with δ fixed, which corresponds to a large overall mass scale compared to the electro-weak scale set by the Higgs expectation value, only one of the angles vanishes,

$$\tan(\psi) \;\longrightarrow\; 0\,. \tag{2.17}$$

The other two parameters are independent of v^2. No matter how high the scale would be, at which the additional couplings are generated, the low energy parameters $\tan(\phi)$ and $\tan(\theta)$ can deviate from SM expressions, since deviations are not suppressed by the high scale.

For the purpose of obtaining a more physical parametrization, it is useful to replace the parameters v, vg_2, vg_Y, M_1, M_2 of the Stueckelberg extended model with those of the SM lagrangian, and fix them through measured quantities, up to the mass scale M. Defining

$$M_Y^2 = \frac{1}{4}g_Y^2 v^2 = \frac{\frac{1}{4}(ev)^2 M_{\mathrm{W}}^2}{M_{\mathrm{W}}^2 - \frac{1}{4}(ev)^2}\,, \tag{2.18}$$

where $v = (\sqrt{2}G_F)^{-1/2}$, we can express M_1 and M_2, or δ and M, in terms of M_{Z}, $M_{\mathrm{Z'}}$, M_{W}, M_Y via

$$\begin{aligned}
M_1^2 &= \frac{M_{\mathrm{Z'}}^2(M_{\mathrm{Z}}^2 - M_{\mathrm{W}}^2) + M_{\mathrm{W}}^2(M_{\mathrm{W}}^2 + M_Y^2 - M_{\mathrm{Z}}^2)}{M_Y^2}\,,\\
M_2^2 &= \frac{(M_{\mathrm{Z'}}^2 - M_{\mathrm{W}}^2 - M_Y^2)(M_{\mathrm{Z}}^2 - M_{\mathrm{W}}^2 - M_Y^2))}{M_Y^2}\,,\\
\frac{M_2^2}{M_1^2} &= \delta^2 = \frac{(M_{\mathrm{Z}}^2 - M_{\mathrm{W}}^2 - M_Y^2)(M_{\mathrm{Z'}}^2 - M_{\mathrm{W}}^2 - M_Y^2)}{M_{\mathrm{Z'}}^2(M_{\mathrm{Z}}^2 - M_{\mathrm{W}}^2) + M_{\mathrm{W}}^2(M_{\mathrm{W}}^2 + M_Y^2 - M_{\mathrm{Z}}^2)}\,.
\end{aligned} \tag{2.19}$$

Now the Stueckelberg lagrangian is fixed by adjusting the parameters to fit the experimental parameters. This requires global fits to the electro-weak data which is outside the scope of this work. If implemented, it should determine the full allowed range of the Stueckelberg parameter space in M_1, M_2. To illustrate the typical values, one convenient choice is to pick $M_{\mathrm{Z'}}$ and δ. Once these are fixed, one can compute the three angles, θ, ϕ, ψ. For instance, for

$$\delta = 0.029\,, \qquad M_{\mathrm{Z'}} = 250\,\mathrm{GeV} \tag{2.20}$$

we find

$$\tan(\phi) = 0.029\,, \qquad \tan(\psi) = 0.002\,, \qquad \tan(\theta) = 0.546\,. \tag{2.21}$$

Note that characteristically, $|\psi| \sim \frac{1}{10}|\phi|$ and θ equals θ_W up to less than a percent.

2.2 Couplings to fermions

Defining a vector of neutral currents $J_a^\mu = (g_X J_X^\mu, g_Y J_Y^\mu, g_2 J_2^{3\mu})$, the couplings to the fermions are easily found by inserting the mass eigenstates into the neutral current (NC) interaction lagrangian

$$\mathcal{L}_{\mathrm{NC}} = g_2 A_\mu^3 J_2^{3\mu} + g_Y B_\mu J_Y^\mu + g_X C_\mu J_X^\mu = \sum_{a=1}^{3} V_{\mu a} J_a^\mu = \sum_{a,b=1}^{3} E_{\mu a}^{[1]} \mathcal{O}_{ba}^{[1]} J_b^\mu\,. \tag{2.22}$$

The three components of this interaction product are easily expressed through the angle parameters,

$$\sum_{b=1}^{3}\mathcal{O}^{[1]}_{ba}J^\mu_b = \begin{bmatrix} \frac{\sin(\psi)}{\sqrt{g_2^2+g_Y^2\cos^2(\phi)}}\left(\cos^2(\phi)g_Y^2 J_Y^\mu - g_2^2 J_2^{3\mu} - \frac{1}{2}\sin(2\phi)g_X g_Y J_X^\mu\right) \\ +\cos(\psi)(\sin(\phi)g_Y J_Y^\mu + \cos(\phi)g_X J_X^\mu) \\ \\ \frac{\cos(\psi)}{\sqrt{g_2^2+g_Y^2\cos^2(\phi)}}\left(\cos^2(\phi)g_Y^2 J_Y^\mu - g_2^2 J_2^{3\mu} - \frac{1}{2}\sin(2\phi)g_X g_Y J_X^\mu\right) \\ -\sin(\psi)(\sin(\phi)g_Y J_Y^\mu + \cos(\phi)g_X J_X^\mu) \\ \\ \frac{g_2 g_Y\cos(\phi)}{\sqrt{g_2^2+g_Y^2\cos^2(\phi)}}\left(J_Y^\mu + J_2^{3\mu} - \frac{g_X}{g_Y}\tan(\phi)J_X^\mu\right) \end{bmatrix}. \tag{2.23}$$

The first line couples to Z′, the second to Z, and the third line is the modified electromagnetic current.

The modification of the current that couples to the photon leads to two effects: First the electric charges of the fields of the SM would get modified. For instance, the charge of the up and the down quark are

$$Q_u = \frac{2}{3} - \frac{g_X}{g_Y}\tan(\phi)Q_X(u)\,, \qquad Q_d = -\frac{1}{3} - \frac{g_X}{g_Y}\tan(\phi)Q_X(d)\,. \tag{2.24}$$

However, the charge neutrality of the neutron requires that $Q_u + 2Q_d = 0$ to very high precision. This, and similar relations for all other fields of the SM, can only be satisfied if the $\mathrm{U}(1)_X$ charges were proportional to their electric charges or vanishing. We, therefore, make the assumption that all fields of the SM itself are neutral under the extra $\mathrm{U}(1)_X$ gauge symmetry, setting $Q_X(\mathrm{SM}) = 0$. This means that the couplings of C_μ with visible matter are strictly forbidden, in order to maintain the charge cancellation between quarks or leptons. On the other hand, there is a priori no such restriction on the matter in the hidden sector to which C_μ can couple. This implies that the masses of charged matter fields in the hidden sector have to be safely outside the current limits of direct detection.

Second, there still is a modification of the electric charge e, the coupling that appears in the term

$$eA^\gamma_\mu J^\mu_{\mathrm{em}} = eA^\gamma_\mu(J_Y^\mu + J_2^{3\mu})\,, \tag{2.25}$$

which is now defined by

$$e = \frac{g_2 g_Y\cos(\phi)}{\sqrt{g_2^2 + g_Y^2\cos^2(\phi)}}\,. \tag{2.26}$$

Thus, the Stueckelberg mechanism effectively changes g_Y to $g_Y\cos(\phi)$. All these modifications, of course, go away, when one takes the SM limit $\delta \to 0$, when $\cos(\phi) \to 1$. Similarly, the standard coupling of the Z-boson,

$$\mathrm{Z}_\mu g_{\mathrm{NC}} J^\mu_{\mathrm{NC}} \;\longrightarrow\; \mathrm{Z}_\mu\frac{1}{\sqrt{g_2^2+g_Y^2}}\left(g_Y^2 J_Y^\mu - g_2^2 J_2^\mu\right) \tag{2.27}$$

is recovered in this limit. One can also read off that the angle ψ takes the role of mixing the couplings of Z and Z′. An important feature of this interaction lagrangian is that the coupling constants of the extra gauge boson are not arbitrary parameters, but uniquely defined through δ and M, the only new parameters of the model (aside from g_X, which we always assume to be of the same order as g_Y or g_2). We postpone a discussion of more experimental properties and concrete signatures of the StSM for later, when we treat the supersymmetric and the ordinary Stueckelberg extension in a combined fashion.

3. The Stueckelberg extension of MSSM

In this section we give the Stueckelberg extension of the minimal supersymmetric standard model (MSSM) [6] which may be labelled the StMSSM. The gauge symmetry is again extended by a single abelian factor $\mathrm{U}(1)_X$, and only the neutral interactions are affected by the Stueckelberg mechanism. As argued above, we now assume that the fields of the MSSM are neutral under the new $\mathrm{U}(1)_X$. Since supersymmetry requires the extra fields to fall into proper multiplets, we add one chiral (or linear) and one vectorsupermultiplet to the MSSM, which combine into a massive spin one multiplet and mix with the other massive vector multiplets after the condensation of the Higgs boson. Beyond the Stueckelberg chiral and vector superfields we in principle also allow for the existence of a hidden sector.

In setting up the supersymmetric extension (using standard superspace notation [14]) we consider the following action for the Stueckelberg chiral multiplet $S = (\rho + i\sigma, \chi, F_S)$

$$\mathcal{L}_{\mathrm{St}} = \int d^2\theta d^2\bar\theta (M_1 C + M_2 B + S + \bar S)^2 \,, \tag{3.1}$$

where $C = (C_\mu, \lambda_C, D_C)$ is the gauge vectormultiplet for $\mathrm{U}(1)_X$, B that for the hyper charge. The supersymmetrized gauge transformations under the new $\mathrm{U}(1)_X$ are

$$\delta_Y B = \Lambda_Y + \bar\Lambda_Y \,, \qquad \delta_Y S = -M_2 \Lambda_Y \,, \tag{3.2}$$

and for the hyper charge

$$\delta_X C = \Lambda_X + \bar\Lambda_X \,, \qquad \delta_X S = -M_1 \Lambda_X \,. \tag{3.3}$$

Although S transforms under the abelian gauge symmetries, it is somewhat misleading to think of it as a charged field in the standard sense of a charged chiral multiplet. To be slightly more specific on our notation, we denote C by

$$C = -\theta\sigma^\mu\bar\theta C_\mu + i\theta\theta\bar\theta\bar\lambda_C - i\bar\theta\bar\theta\theta\lambda_C + \frac{1}{2}\theta\theta\bar\theta\bar\theta D_C \,. \tag{3.4}$$

Similarly for B with B_μ, λ_B and D_B, and S is given by

$$S = \frac{1}{2}(\rho + i\sigma) + \theta\chi + i\theta\sigma^\mu\bar\theta\frac{1}{2}(\partial_\mu\rho + i\partial_\mu\sigma) + \theta\theta F_S + \frac{i}{2}\theta\theta\bar\theta\bar\sigma^\mu\partial_\mu\chi + \frac{1}{8}\theta\theta\bar\theta\bar\theta(\Box\rho + i\Box\sigma) \,. \tag{3.5}$$

Its scalar component contains the scalar ρ and the axionic pseudo-scalar σ. This leads to [16, 17]

$$\begin{aligned} \mathcal{L}_{\mathrm{St}} = & -\frac{1}{2}(M_1 C_\mu + M_2 B_\mu + \partial_\mu\sigma)^2 - \frac{1}{2}(\partial_\mu\rho)^2 - i\chi\sigma^\mu\partial_\mu\bar\chi + 2|F_S|^2 + \\ & +\rho(M_1 D_C + M_2 D_B) + [\chi(M_1\lambda_C + M_2\lambda_B) + \mathrm{h.c.}] \,. \end{aligned} \tag{3.6}$$

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For the gauge fields we add the standard kinetic terms

$$\mathcal{L}_{\rm gkin} = -\frac{1}{4}C_{\mu\nu}C^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - i\lambda_B\sigma^\mu\partial_\mu\bar\lambda_B - i\lambda_C\sigma^\mu\partial_\mu\bar\lambda_C + \frac{1}{2}D_C^2 + \frac{1}{2}D_B^2\,.$$

For the matter fields, chiral superfields Φ_i and $\Phi_{{\rm hid},i}$ are introduced. The fermions (quarks q_i, leptons l_i, Higgsinos $\tilde h_i$) of the MSSM will be collectively denoted as f_i, hidden sector fermions as $f_{{\rm hid},i}$. The scalars (sfermions $\tilde q_i$, sleptons $\tilde l_i$ and the two Higgs fields h_i) are summarized as z_i and $z_{{\rm hid},i}$.[6] The lagrangian reads

$$\mathcal{L}_{\rm matt} = \int d^2\theta d^2\bar\theta\left[\sum_i\bar\Phi_i e^{2g_YQ_YB+2g_XQ_XC}\Phi_i + \sum_i\bar\Phi_{{\rm hid},i}e^{2g_YQ_YB+2g_XQ_XC}\Phi_{{\rm hid},i}\right].$$

where $Q_Y = Y/2$, and where Y is the hyper charge so that $Q = T_3 + Y/2$. As mentioned already, the SM matter fields do not carry any charge under the hidden gauge group, i.e. $Q_X\Phi_i = 0$. Thus we have

$$\begin{aligned}\mathcal{L}_{{\rm matt},i} &= -|D_\mu z_i|^2 - if_i\sigma^\mu\partial_\mu\bar f_i + |F_i|^2 + g_YB_\mu J^\mu_{Yi} + g_XC_\mu J^\mu_{Xi} - \\ &\quad -\sqrt{2}[ig_YQ_Yz_i\bar f_i\bar\lambda_B + ig_XQ_Xz_i\bar f_i\bar\lambda_C + {\rm h.c.}] + \\ &\quad +g_YD_B(\bar z_iQ_Yz_i) + g_XD_C(\bar z_iQ_Xz_i)\,, \end{aligned} \tag{3.7}$$

where $D_\mu = \partial_\mu + ig_YQ_YB_\mu + ig_XQ_XC_\mu$, and

$$J^\mu_{Yi} = f_iQ_Y\sigma^\mu\bar f_i\,, \qquad J^\mu_{Xi} = f_iQ_X\sigma^\mu\bar f_i\,. \tag{3.8}$$

The above uses standard notation with Weyl spinors. It is convenient before passing to mass eigenstates to define now Majorana spinors in the form

$$\psi_S = \begin{pmatrix}\chi_\alpha\\ \bar\chi^{\dot\alpha}\end{pmatrix}, \qquad \lambda_X = \begin{pmatrix}\lambda_{C\alpha}\\ \bar\lambda_C^{\dot\alpha}\end{pmatrix}, \qquad \lambda_Y = \begin{pmatrix}\lambda_{B\alpha}\\ \bar\lambda_B^{\dot\alpha}\end{pmatrix}. \tag{3.9}$$

Thus the Stueckelberg extension introduces two new Majorana spinors in the system, i.e. ψ_S and λ_X. We also rewrite the matter fermions in terms of Majorana fields, but still use the same symbols f_i here, as before for the Weyl fermions. One has for instance the following identities

$$\begin{aligned}\chi\lambda_C + \bar\chi\bar\lambda_C &= \bar\psi_S\lambda_X\,,\\ \chi\lambda_C - \bar\chi\bar\lambda_C &= \bar\psi_S\gamma_5\lambda_X\,,\\ \chi\sigma^\mu\partial_\mu\bar\chi - (\partial_\mu\chi)\sigma^\mu\bar\chi &= \bar\psi_S\gamma^\mu\partial_\mu\psi_S\,.\end{aligned} \tag{3.10}$$

We may then write the total lagrangian (by substituting back in the values for D_B and D_C) in the form

$$\mathcal{L}_{\rm St} + \mathcal{L}_{\rm gkin} + \mathcal{L}_{{\rm matt},i} = -\frac{1}{2}(M_1C_\mu + M_2B_\mu + \partial_\mu\sigma)^2 - \frac{1}{2}(\partial_\mu\rho)^2 - \frac{1}{2}(M_1^2 + M_2^2)\rho^2 -$$

[6] The matter chiral multiplets are defined exactly according to the conventions of [14], while S carries some extra factors for convenience.

$$
\begin{aligned}
&-\frac{i}{2}\bar{\psi}_S\gamma^\mu\partial_\mu\psi_S - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}C_{\mu\nu}C^{\mu\nu} - \frac{i}{2}\bar{\lambda}_Y\gamma^\mu\partial_\mu\lambda_Y - \\
&-\frac{i}{2}\bar{\lambda}_X\gamma^\mu\partial_\mu\lambda_X - |D_\mu z_i|^2 - \frac{i}{2}\bar{f}_i\gamma^\mu\partial_\mu f_i + \frac{1}{2}g_Y B_\mu \bar{f}_i\gamma^\mu Q_Y f_i + \\
&+\frac{1}{2}g_X C_\mu \bar{f}_i\gamma^\mu Q_X f_i + M_1\bar{\psi}_S\lambda_X + M_2\bar{\psi}_S\lambda_Y - \\
&-\sqrt{2}g_X[iz_iQ_X\bar{f}_i\lambda_X + \text{h.c.}] - \rho(g_Y M_2(\bar{z}_iQ_Y z_i)+g_X M_1(\bar{z}_iQ_X z_i)) - \\
&-\frac{1}{2}\Big[\sum_i \bar{z}_i g_Y Q_Y z_i\Big]^2 - \frac{1}{2}\Big[\sum_i \bar{z}_i g_X Q_X z_i\Big]^2 . \qquad (3.11)
\end{aligned}
$$

Of course, one has to add the hidden sector fields and sum over i when appropriate. We have already pointed out that the MSSM itself is neutral under the new gauge symmetry $\mathrm{U}(1)_X$, but the hidden sector fields may well be charged under it. In order to get a model that represents the pure Stueckelberg effect, we further let the hidden sector be neutral under the gauge group of the SM, i.e. we really demand it to be hidden with respect to the SM gauge interactions.

The modifications that are introduced by the Stueckelberg extension are now completely evident: We have added the degrees of freedom of one abelian gauge vector multiplet, the vector field C_μ and its gaugino λ_X, as well as the chiral multiplet with the complex scalar $\rho + ia$ and the fermion ψ_S. There are three channels for the new sector to communicate to the SM fields: *i*) the mixing of neutral gauge bosons through the non-diagonal vector boson mass matrix, just as in the Stueckelberg extension of the SM, *ii*) the mixing of neutralinos through the fermion mass matrix with the off-diagonal terms involving the gauginos and ψ_S, *iii*) the cubic couplings of ρ with the scalar partners of SM fermions and the Higgs bosons.

Through the Stueckelberg coupling, a combination of the vector fields B_μ and C_μ gets a mass, and absorbs the axionic component σ as its longitudinal mode. The real part ρ gets a mass M. We shall see that mass eigenstates that combine out of the two gauginos and ψ_S will just form a massive fermion of identical mass as the vector and the scalar. Thus, out of the massless two vector and one chiral multiplet, one massive spin one (out of a vector, a Dirac fermion and a scalar) and one massless vector multiplet are combined. When the Higgs condensate is introduced, the massless vector multiplet will mix with the 3-component of the adjoint $\mathrm{SU}(2)_L$ gauge boson multiplet.

3.1 Adding soft supersymmetry breaking terms

Including soft supersymmetry breaking terms will finally break up the mass degeneracy of the spectrum. The soft breaking terms relevant for the further discussion are

$$
\mathcal{L}_{\text{soft}} = -\frac{1}{2}m_\rho^2\rho^2 - \frac{1}{2}\tilde{m}_Y\bar{\lambda}_Y\lambda_Y - \frac{1}{2}\tilde{m}_X\bar{\lambda}_X\lambda_X - m_1^2|h_1|^2 - m_2^2|h_2|^2 - m_3^2(h_1\cdot h_2 + \text{h.c.}) , \quad (3.12)
$$

with $m_1^2 = m_{h_1}^2 + |\mu|^2$, $m_2^2 = m_{h_2}^2 + |\mu|^2$, $m_3^2 = |\mu B|$, where μ is the Higgs mixing parameter (which is not really soft but part of the superpotential) and B is the soft bilinear coupling. Note that there is no soft mass for the chiral fermion ψ_S.

3.2 Adding Fayet-Illiopoulos terms

The above analysis was so far without the Fayet-Illiopoulos (FI) terms. In the present case it means that one has the freedom to introduce two terms in the lagrangian of the form

$$\mathcal{L}_{\rm FI} = \xi_B D_B + \xi_C D_C \,. \tag{3.13}$$

For the contribution of ξ_B we make the usual assumption that it is subdominant and can be neglected in the Higgs potential that drives spontaneous gauge symmetry breaking. This remains true for the modified field

$$-D_B = \xi_B + M_2\rho + g_Y \sum_i \bar{z}_i Q_Y z_i \,, \tag{3.14}$$

as it will turn out that the modification $M_2\rho$ will be very small. For the FI-term with ξ_C one finds on eliminating the auxiliary field D_C

$$-D_C = \xi_C + M_1\rho + g_X \sum_i \bar{z}_i Q_X z_i \,. \tag{3.15}$$

The modification of eq. (3.11) in the presence of FI terms is implemented by the replacement

$$\begin{aligned} \sum_i \bar{z}_i g_Y Q_Y z_i &\rightarrow \xi_B + \sum_i \bar{z}_i g_Y Q_Y z_i \,, \\ \sum_i \bar{z}_i g_X Q_X z_i &\rightarrow \xi_C + \sum_i \bar{z}_i g_X Q_X z_i \end{aligned} \tag{3.16}$$

in eq. (3.11). Since we assume that the charges Q_X of the MSSM fields are all vanishing, this will not have any impact on the visible sector mass or quartic couplings. Depending on the charges of the hidden sector field, such a FI-term may be able to drive a spontaneous breaking of the U(1)$_X$ gauge symmetry, which would result in a mass term for the photon mass eigenstate, and thus has to be excluded.

4. Implications and predictions

Here we now discuss the consequences of the extensions of the SM or MSSM with an extra U(1)$_X$ that couples to a pseudo-scalar σ, together with the hyper charge gauge boson multiplet, in the way of the Stueckelberg mechanism. First, we shall go through the modifications of the SM. They all refer to the non-diagonal mass squared matrix of the neutral gauge bosons, and the effects of their mixing. These effects will also be reproduced in the MSSM without any modification, which then contains further signatures through the modified neutral scalar and neutral fermion sectors.

4.1 Comparison to the standard model

Some of the implications of the extended model have already been explained above. Roughly speaking, diagonalizing the mass squared matrix of the neutral gauge bosons introduces a mixing of all three vector fields, and of the currents they couple to. For the photon this implies that the coupling constant to the electromagnetic current is modified, and that it may couple to hidden sector matter charged under $\mathrm{U}(1)_X$. The latter is a very interesting phenomenon, since it may give indirect evidence of hidden sector matter, which is otherwise invisible to gauge interactions. However, the couplings to the hidden sector are highly model dependent and could even be completely suppressed as discussed at the end of section 7. For the neutral current interactions, the mixing also implies a change of coupling constants and currents, and a coupling to hidden matter. The latter may not be so dramatic here, since the interactions are only short-ranged.

4.1.1 Neutral current interactions: ρ parameters

A useful parameter to study the neutral current interactions is the conventional ρ parameter which is defined as the ratio in the effective low energy lagrangian of the neutral and the charged current interactions.[7] For the SM at the tree level this ratio is

$$\rho_{\mathrm{SM}} = \frac{M_{\mathrm{W}}}{\cos(\theta_W) M_{\mathrm{Z}}}\,, \tag{4.1}$$

and there are small deviation from unity due to radiative corrections. For the model at hand this issue is more complicated, and the neutral and charged current interactions can no longer be compared with just one ratio, because there are now two neutral massive gauge bosons. To see this, we can eliminate Z- and Z′-bosons at low energy to obtain an effective neutral current interaction, which we can write as follows

$$\begin{aligned} \mathcal{L}_{\mathrm{NC-eff}} = \frac{4G_F}{\sqrt{2}} 2 \Big[&\rho_{\mathrm{Z}} (J_3^\mu - \sin^2(\theta_{\mathrm{Z}}) J_{\mathrm{em}}^\mu)(J_{3\mu} - \sin^2(\theta_{\mathrm{Z}}) J_{\mathrm{em}\mu}) + \\ &+ \rho_{\mathrm{Z}'} (J_3^\mu - \sin^2(\theta_{\mathrm{Z}'}) J_{\mathrm{em}}^\mu)(J_{3\mu} - \sin^2(\theta_{\mathrm{Z}'}) J_{\mathrm{em}\mu}) \Big]\,, \end{aligned} \tag{4.2}$$

while the charged effective current-current lagrangian is unchanged

$$\mathcal{L}_{\mathrm{CC-eff}} = \frac{4G_F}{\sqrt{2}} J_\mu^+ J^{-\mu}\,. \tag{4.3}$$

Above, we have defined ρ_{Z} and $\rho_{\mathrm{Z}'}$ by

$$\rho_{\mathrm{Z}} = \frac{M_{\mathrm{W}}^2 f_{\mathrm{Z}}^2}{M_{\mathrm{Z}}^2 \cos(\theta)}\,, \qquad \rho_{\mathrm{Z}'} = \frac{M_{\mathrm{W}}^2 f_{\mathrm{Z}'}^2}{M_{\mathrm{Z}'}^2 \cos(\theta)} \tag{4.4}$$

and the effective decay constants f_{Z} and $f_{\mathrm{Z}'}$ by

$$f_{\mathrm{Z}} = \cos(\psi) + \sin(\theta)\tan(\phi)\sin(\psi)\,, \qquad f_{\mathrm{Z}'} = \sin(\psi) + \sin(\theta)\tan(\phi)\cos(\psi)\,. \tag{4.5}$$

Finally θ_{Z} and $\theta_{\mathrm{Z}'}$ are defined by

$$\begin{aligned} \sin^2(\theta_{\mathrm{Z}}) &= \frac{\sin^2(\theta) - \sin(\theta)\tan(\phi)\tan(\psi)}{1 + \sin(\theta)\tan(\phi)\tan(\psi)}\,, \\ \sin^2(\theta_{\mathrm{Z}'}) &= \frac{\sin^2(\theta)\tan(\psi) + \sin(\theta)\tan(\phi)}{\tan(\psi) + \sin(\theta)\tan(\phi)}\,. \end{aligned} \tag{4.6}$$

[7]The ρ parameters discussed here should not be confused with the scalar field ρ that appears in the Stueckelberg extension of the MSSM.

Further we can also define a parameter ρ analogous to the conventional parameter in the SM so that $\rho = M_{\rm W}/(M_{\rm Z}\cos(\theta))$. Even at the tree-level $\rho_{\rm Z}, \rho_{\rm Z'}$, and ρ are all different. Further, eq. (4.2) shows that in the present model different combinations of $\rho_{\rm Z}$ and $\rho_{\rm Z'}$ appear for the operators $J_2^3 \cdot J_2^3$, $J_{\rm em} \cdot J_{\rm em}$, and $J_2^3 \cdot J_{\rm em}$. Thus the relevant ratio of charged and neutral interaction strength will depend on the process. In the limit that $M_2 = 0$ one has $\rho_{\rm Z'} = 0$ and $\rho_{\rm Z} = \rho = \rho_{\rm SM}$.

Currently, there are stringent constraints on the neutral current processes and the data is consistent with the SM. However, the error corridor in the experimental measurements allow the possibility of new physics including the possibility of new Z′ bosons and this topic has been investigated extensively in the literature. This possibility also applies to the current model if the contribution of the new sector is sufficiently small to be consistent with the experimental error corridor. Thus, for example, for $\psi \sim 1^0, \phi \sim 1^0$, and setting $\sin^2(\theta) = \sin^2(\theta_W) = 0.23$ one finds, $\sin^2(\theta_{\rm Z}) = 0.2298$, and $1 - \rho_{\rm Z}/\rho_{\rm SM} = 0.0001$, while $\rho_{\rm Z'}/\rho_{\rm SM} = 0.025 \times M_{\rm Z}^2/M_{\rm Z'}^2$, which gives $\rho_{\rm Z'}/\rho_{\rm SM} \sim 0.0025$ for $M_{\rm Z'}/M_{\rm Z} = 3$. These are consistent with the current error corridors on ρ, of the order of 0.005.

4.1.2 Visible width and branching ratios of Z′

In greater detail, one can write the couplings of the first generation as follows

$$\begin{aligned}
\mathcal{L}_{{\rm Z}l\bar{l}} = &-\frac{1}{2}\left[-\frac{g_2^2 - \cos^2(\phi) g_Y^2}{\sqrt{g_2^2 + \cos^2(\phi) g_Y^2}}\cos\psi - \sin(\phi)\sin(\psi) g_Y\right]\bar{e}_L\gamma^\mu e_L {\rm Z}_\mu - \\
&-\left[\frac{\cos^2(\phi) g_Y^2}{\sqrt{g_2^2 + \cos^2(\phi) g_Y^2}}\cos(\psi) - \sin(\phi)\sin(\psi) g_Y\right]\bar{e}_R\gamma^\mu e_R {\rm Z}_\mu - \\
&-\frac{1}{2}\left[\sqrt{g_2^2 + \cos^2(\phi) g_Y^2}\cos(\psi) - \sin(\phi)\sin(\psi) g_Y\right]\bar{\nu}_e\gamma^\mu \nu_e {\rm Z}_\mu - \\
&-\frac{1}{2}\left[-\frac{g_2^2 - \cos^2(\phi) g_Y^2}{\sqrt{g_2^2 + \cos^2(\phi) g_Y^2}}\sin(\psi) + \sin(\phi)\cos(\psi) g_Y\right]\bar{e}_L\gamma^\mu e_L {\rm Z}'_\mu - \\
&-\left[\frac{\cos^2(\phi) g_Y^2}{\sqrt{g_2^2 + \cos^2(\phi) g_Y^2}}\sin(\psi) + \sin(\phi)\cos(\psi) g_Y\right]\bar{e}_R\gamma^\mu e_R {\rm Z}'_\mu - \\
&-\frac{1}{2}\left[\sqrt{g_2^2 + \cos^2(\phi) g_Y^2}\sin(\psi) + \sin(\phi)\cos(\psi) g_Y\right]\bar{\nu}_e\gamma^\mu \nu_e {\rm Z}'_\mu\,.
\end{aligned} \tag{4.7}$$

And the couplings of Z and Z′ with quarks are given by

$$\begin{aligned}
\mathcal{L}_{{\rm Z}q\bar{q}} = &-\sqrt{g_2^2 + \cos^2(\phi) g_Y^2}\,\times \\
&\times\Big[{\rm Z}_\mu\left(J_3^\mu - \sin^2(\theta) J_{\rm em}^\mu\right)\cos(\psi) + {\rm Z}_\mu (J_{\rm em}^\mu - J_3^\mu)\sin(\theta)\tan(\phi)\sin(\psi) + \\
&+ {\rm Z}'_\mu (J_3^\mu - \sin^2(\theta) J_{\rm em}^\mu)\sin(\psi) - {\rm Z}'_\mu (J_{\rm em}^\mu - J_3^\mu)\sin(\theta)\tan(\phi)\cos(\psi)\Big]\,.
\end{aligned} \tag{4.8}$$

In addition to the new couplings of the quarks to the ${\rm Z}'_\mu$ boson the couplings of ${\rm Z}_\mu$ with quarks are also affected. Below we give a comparison of the decay branching ratios for the

Ratio of branching ratios	Z decay	Z′ decay
$l\bar{l}/\nu\bar{\nu}$	0.5	5
$b\bar{b}/\tau\bar{\tau}$	$(3-4s_W^2+\frac{8}{3}s_W^4)/(1-4s_W^2+8s_W^4)$	$\frac{1}{3}$
$u\bar{u}/d\bar{d}$	$(3-8s_W^2+\frac{32}{3}s_W^4)/(3-4s_W^2+\frac{8}{3}s_W^4)$	$\frac{17}{5}$

Table 1: A comparison of the ratio of branching ratios into quarks and leptons Z′ versus Z ($s_W = \sin(\theta_W)$).

decay of the Z′ into quarks and leptons versus the branching ratios for the decay of the Z into quarks and leptons. We display the results for $|\psi| \ll |\phi|$ in table 1.

In the same approximation the total decay width of Z′ into the visible sector quarks and leptons is given by

$$\Gamma\left(\mathrm{Z}' \to \sum_i f_i\bar{f}_i\right) \simeq M_{\mathrm{Z}'} g_Y^2 \sin^2(\phi) \times \begin{cases} \frac{103}{288\pi} & \text{for } M_{\mathrm{Z}'} < 2m_t \\ \frac{5}{12\pi} & \text{for } M_{\mathrm{Z}'} > 2m_t \end{cases} \tag{4.9}$$

The decay signatures of the Z′ boson are very different from those of the Z boson of the SM. The reason for this difference arises from the fact that the Z′ dominantly decays via the couplings proportional to g_Y as can be seen by making the approximation $|\psi| \ll |\phi| \ll 1$ in eq. (4.7) and eq. (4.8).

4.2 The bosonic sector of the extended MSSM

The bosonic sector of the StMSSM consists of the neutral vector bosons, the Stueckelberg scalar ρ, the Higgs fields and the sfermions of the MSSM. The Stueckelberg axion σ is decoupled after gauge fixing, and is absorbed by the gauge bosons. The analysis of the mass matrix of the vector bosons remains unchanged from that of the SM as discussed in section 2 and we do not have to repeat it here.

We have already mentioned the assumptions that go into the definitions of the model. We take all the matter fields of the MSSM and the two Higgs multiplets to be neutral under the $\mathrm{U}(1)_X$, and we also demand that there is no charged scalar condensate formed in the hidden sector, e.g. no vacuum expectation value $\langle \bar{z}_i Q_X z_i \rangle \neq 0$. This would add another term to the mass matrix (2.7) and finally give a mass to the photon eigenstate. We, therefore, impose $\langle z_i \rangle = 0$ for all hidden scalars z_i that carry charge under $\mathrm{U}(1)_X$, which are the only ones relevant for us.

Under these assumptions, the subsector of the StMSSM which contains the neutral vector bosons, and their couplings to the conserved currents is just identical to the StSM. We are left in the bosonic sector with the extra neutral scalar ρ, that mixes with the neutral components of the Higgs doublets.

4.2.1 The scalar Higgs fields and the Stueckelberg scalar ρ

The scalar potential for the two Higgs-doublets of the MSSM plus the Stueckelberg scalar ρ involves a non-diagonal mass squared matrix, similar to the mixing of neutral gauge bosons. As explained in the previous section, the Higgs fields are neutral under $\mathrm{U}(1)_X$,

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hidden sector fields are neutral under hyper charge, and there are no condensates charged under $\mathrm{U}(1)_X$. Then we get

$$\begin{aligned}\mathcal{V}(h_1,h_2,\rho) = {} & \left(m_1^2 - \frac{1}{2}\rho g_Y M_2\right)|h_1|^2 + \left(m_2^2 + \frac{1}{2}\rho g_Y M_2\right)|h_2|^2 + m_3^2(h_1\cdot h_2 + \text{h.c.}) + \\ & + \frac{g_2^2+g_Y^2}{8}|h_1|^4 + \frac{g_2^2+g_Y^2}{8}|h_2|^4 + \frac{g_2^2-g_Y^2}{4}|h_1|^2|h_2|^2 - \frac{g_2^2}{2}|h_1\cdot h_2|^2 + \\ & + \frac{1}{2}(M_1^2+M_2^2+m_\rho^2)\rho^2\,. \end{aligned} \tag{4.10}$$

The Higgs doublets are defined $h_1 = (h_1^0, h_1^-)^T$, $h_2 = (h_2^+, h_2^0)^T$, and $h_1\cdot h_2 = h_1^0 h_2^0 - h_1^- h_2^+$. For the Higgs scalars h_1^0, h_2^0, and for ρ we make replacements

$$h_1^0 \to \frac{1}{\sqrt{2}}(v_1 + h_1^0)\,, \qquad h_2^0 \to \frac{1}{\sqrt{2}}(v_2 + h_2^0)\,, \qquad \rho \to v_\rho + \rho\,, \tag{4.11}$$

where v_i and v_ρ are the vacuum expectation values, and assumed to be real. As usual, they are parameterized by

$$v_1 = v\cos(\beta)\,, \qquad v_2 = v\sin(\beta)\,. \tag{4.12}$$

For ρ one has

$$v_\rho = \frac{2 g_Y M_{\mathrm{W}}^2 M_2}{g_2^2 M_\rho^2}\cos(2\beta)\,, \tag{4.13}$$

where $M_\rho^2 = M^2 + m_\rho^2 = M_1^2 + M_2^2 + m_\rho^2$. To give a rough estimate for large $\tan(\beta)$, one has $|g_Y M_2 v_\rho| \sim 10^{-5} M_{\mathrm{W}}^2$.

Substituting v_ρ back into the potential adds an extra contribution to the Higgs potential. The minimization of the effective potential with respect to the h_1^0 and h_2^0 gives two conditions and one combination of these is affected by ρ, and one has

$$\frac{1}{2}M_0^2 = \frac{m_1^2 - m_2^2\tan^2(\beta)}{\tan^2(\beta)-1} + \frac{g_Y M_2 v_\rho}{2\cos(2\beta)}\,. \tag{4.14}$$

where $M_0^2 = (g_2^2+g_Y^2)v^2/4$, so that M_0 coincides with the Standard Model tree level prediction in the limit when the Stueckelberg effects vanish. In the absence of the Stueckelberg effect on has $v_\rho = 0$ and one recovers the well known result of radiative breaking of the electro-weak symmetry in SUGRA models [18]. We see now that the Stueckelberg effect modifies the equation that determines $M_0^2 \sim M_Z^2$, but only by a tiny correction.

Inserting the vacuum expectation values back into the potential, we compute now the mass matrix for the neutral Higgs fields. The CP-odd neutral Higgs is not affected by the Stueckelberg extension. However, in the CP-even sector one has three states, i.e. $\phi_1 = \mathrm{Re}(h_1^0), \phi_2 = \mathrm{Re}(h_2^0)$, and ρ, which mix. The coupling of the h_i^0 to ρ adds off-diagonal bilinear interactions $h_i\rho$. In terms of the basis $S_a = (\phi_1,\phi_2,\rho)_a^T$ for the CP-even neutral scalars, the mass term reads

$$-\frac{1}{2}\sum_{a,b=1}^{3} S_a M_{ab}^{[0]2} S_b \tag{4.15}$$

with the following mass matrix (using the upper index [0] for spin 0)

$$M^{[0]2}_{ab} = \begin{bmatrix} M_0^2 c_\beta^2 + m_A^2 s_\beta^2 & -(M_0^2 + m_A^2) s_\beta c_\beta & -\frac{1}{2} g_Y M_2 v c_\beta \\ -(M_0^2 + m_A^2) s_\beta c_\beta & M_0^2 s_\beta^2 + m_A^2 c_\beta^2 & \frac{1}{2} g_Y M_2 v s_\beta \\ -\frac{1}{2} g_Y M_2 v c_\beta & \frac{1}{2} g_Y M_2 v s_\beta & M_\rho^2 \end{bmatrix}_{ab} , \tag{4.16}$$

where $(s_\beta, c_\beta) = (\sin(\beta), \cos(\beta))$. The eigenstates we denote by

$$E^{[0]}_a = (H^0_1, H^0_2, H^0_3)^T_a \,, \tag{4.17}$$

and arrange them so that

$$E^{[0]}_a \longrightarrow (H^0, h^0, \rho)^T_a \,, \tag{4.18}$$

when $\delta \to 0$. Then h^0 is the light neutral Higgs of the MSSM and H^0 is the heavy one. Instead of two, we now have three neutral Higgs states, all of which are CP-even states in resonant production in the $q\bar{q}$ channel. In CP-violating channels the number of states will increase to four, since the above three CP-even states will mix with the CP-odd state A^0. The effect of the mixing on the mass eigenvalues of h^0 and H^0 is governed roughly by the ratios $g_Y M_2 v / m_i^2$, for $m_i = M_0, M_\rho, m_A$, and is model-dependent. The correction on the lightest Higgs boson mass could be either positive or negative. For example, it turns out negative when $\tan(\beta)$ is large and $M_\rho > M_0$. The size of the correction could be as large as a few GeV but is significantly smaller than the loop corrections. A quantitative analysis requires a global fit to the electro-weak data and is beyond the scope of the present work.

The new state that appears above is H^0_3 which has the quantum numbers $J^{\rm CP} = 0^+$. This state is mostly the ρ state and its decay into visible sector will be dominantly into $t\bar{t}$, provided $m_{H^0_3} > 2m_t$, or otherwise into $b\bar{b}$. We expect the size of the relevant mixing parameter to be $\mathcal{O}(M_2/M_1) \sim 0.01$ and thus the decay width will be in the range of MeV or less. The production of such a resonance in e^+e^- colliders will be difficult since the couplings of this state to fermions is proportional to the mass and in addition there are suppression factors. A possible production mechanism is at hadron colliders via the Drell-Yan process using the $q\bar{q}H_3$ vertex, where the largest contributions will arise when $q = (b, t)$.

4.2.2 Stueckelberg corrections to sfermion masses

The Stueckelberg effect modifies the D-term correction to squark and slepton masses. This can be seen by examining the effective lagrangian after elimination of D_B and D_C. The effective potential then is

$$\mathcal{V}(\tilde{q}_i, \tilde{l}_i, \rho) = \frac{1}{2}\left[\sum_i \bar{z}_i \frac{g_Y Y}{2} z_i\right]^2 + \rho M_2 \sum_i \left(\bar{z}_i \frac{g_Y Y}{2} z_i\right) . \tag{4.19}$$

The D-term correction to the mass of the sfermion z_i is

$$\Delta \tilde{m}^2_{z_i} = \frac{Y_i}{2} v_\rho g_Y M_2 + \frac{Y_i}{2} \sin^2(\theta_W) \cos(2\beta) M_0^2 \,. \tag{4.20}$$

Of course, to the above we must add the D-term correction from $\mathrm{SU}(2)_L$ sector. Finally, we note that an interesting sum rule results in the case when $M_2/M_1 \ll 1$ relating the ρ mass and the Z′ mass. In this limit one finds from eq. (2.8) and eq. (4.16) the following approximate sum rule

$$M_\rho^2 \simeq M_{Z'}^2 + m_\rho^2 \,. \tag{4.21}$$

Clearly, $M_\rho \geq M_{Z'}$, the additional spin zero state is heavier than the Z′ boson.

4.3 The fermionic sector of the extended MSSM

We discuss now the fermionic sector of the theory. For the neutral fermions instead of four neutral Majorana fields in the MSSM, we have a set of six fields. These consist of the three gauginos, the two Higgsinos $\tilde{h}_i$, and the extra Stueckelberg fermions ψ_S. We order the six neutral fields into a vector ψ

$$\psi_a = (\psi_S, \lambda_X, \lambda_Y, \lambda_3, \tilde{h}_1, \tilde{h}_2)_a^T \tag{4.22}$$

and write the mass term as (upper index [1/2] for spin 1/2)

$$-\frac{1}{2}\sum_{a,b=1}^{6} \bar{\psi}_a M_{ab}^{[1/2]} \psi_b \,. \tag{4.23}$$

From the Stueckelberg correction to the MSSM lagrangian, the only correction is due to the coupling of ψ_S to gauginos, because the triliniear coupling with the scalars z_i does not induce bilinear fermion interactions, as $\langle z_i \rangle = 0$. After spontaneous breaking of the electro-weak symmetry the neutralino mass matrix in the above basis is given by

$$M_{ab}^{[1/2]} = \begin{bmatrix} 0 & M_1 & M_2 & 0 & 0 & 0 \\ M_1 & \tilde{m}_S & 0 & 0 & 0 & 0 \\ M_2 & 0 & \tilde{m}_1 & 0 & -c_\beta s_W M_0 & s_\beta s_W M_0 \\ 0 & 0 & 0 & \tilde{m}_2 & c_\beta c_W M_0 & -s_\beta c_W M_0 \\ 0 & 0 & -c_\beta s_W M_0 & c_\beta c_W M_0 & 0 & -\mu \\ 0 & 0 & s_\beta s_W M_0 & -s_\beta c_W M_0 & -\mu & 0 \end{bmatrix}_{ab} , \tag{4.24}$$

We note that the zero entry in the upper left hand corner arises due to the Weyl fermions not acquiring soft masses. The above gives rise to six Majorana mass eigenstates which we label as follows

$$E_a^{[1/2]} = (\chi_1^0, \chi_2^0, \chi_3^0, \chi_4^0, \chi_5^0, \chi_6^0)_a^T \,. \tag{4.25}$$

The two additional Majorana eigenstates χ_5^0, χ_6^0 are due to the Stueckelberg extension. To get an idea of the effect of the Stueckelberg sector, we exhibit the eigenvalues in the limit when M_Z is negligible relative to all other mass parameters in the mass matrix. In this case the spectrum consists of

$$m_{\chi_i^0}(i=1-4)\,, \qquad m_{\chi_5^0} = \sqrt{M_1^2 + \frac{1}{4}\tilde{m}_S^2} + \frac{1}{2}\tilde{m}_S\,, \qquad m_{\chi_6^0} = \sqrt{M_1^2 + \frac{1}{4}\tilde{m}_S^2} - \frac{1}{2}\tilde{m}_S \tag{4.26}$$

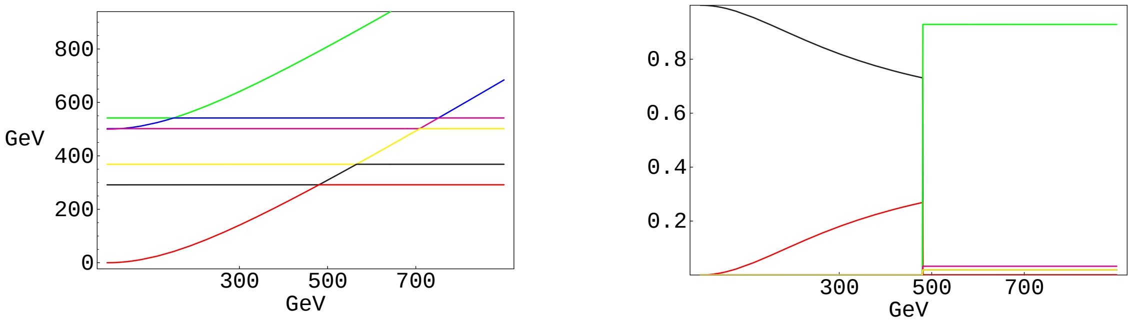

Figure 1: Plot of the neutralino mass spectrum as a function of M (left), for values $\tan(\beta) = 3$, $\mu = 500$, $m_2 = 400$, $m_1 = 300$, $m_S = 500$, $\delta = 0.029$ and of the (squared) components of the LSP also as a function of M (right).

where $m_{\chi_i^0}$ $(i = 1-4)$ are the four eigenvalues that arise from diagonalization of the 4×4 mass matrix in the lower right hand corner. These are the usual eigenvalues that one has in the MSSM. The eigenvalues $m_{\chi_5^0}$ and $m_{\chi_6^0}$ correspond to the heavy and light additional states which we christen as Stueckelberginos. For the case when $m_{\chi_{5,6}^0} > m_{\chi_1^0}$ not much will change, and the analysis of dark matter will essentially remain unchanged. However, for the case when the light Stueckelbergino is lighter than the lightest of $m_{\chi_i^0}$ $(i = 1-4)$, then the situation is drastically changed. In this case the lightest supersymmetric particle (LSP) is no longer a neutralino, i.e. of the set $m_{\chi_i^0}$ $(i = 1-4)$, but rather the Stueckelbergino $\chi_{\rm St}^0 = \chi_6^0$.

We illustrate the above phenomena in figure 1. In the left part of this figure the masses of the six neutralinos are plotted as a function of M for the inputs given there. For values of M above around 500 GeV the LSP is the usual MSSM neutralino and its mass is essentially unaffected by M. However, as we move to values of M below 500 GeV, one finds that there is a sudden transition and the LSP becomes mostly a Stueckelberg fermion and its mass then varies rapidly with M. The same phenomenon is illustrated in the figure to the right where the square of the magnitudes of the components of the LSP in spectral decomposition are plotted, i.e. one writes the LSP (χ^0) as follows

$$\chi^0 = C_S\psi_S + C_X\lambda_X + C_Y\lambda_Y + C_3\lambda_3 + C_1\tilde{h}_1 + C_2\tilde{h}_2\,. \tag{4.27}$$

Thus, on the right hand side figure 1 below $M = 500$ GeV the upper curve is $|C_S|^2$ and the lower curve is $|C_X|^2$ while the other components are too small to be visible. Above $M = 500$ GeV, the upper curve is $|C_Y|^2$ while the next lower curve is $|C_3|^2$ etc. Again in this figure we see a rather sudden transition from the LSP being an almost pure MSSM particle above $M = 500$ GeV to being an almost Stueckelberg fermion below $M = 500$ GeV. Another view of the same phenomenon is given in figure 2 in a plot showing the LSP mass along the vertical axis versus values of M and δ along the horizontal axes. It displays the very weak dependence of the mass of the lightest neutralino eigenstate on δ over basically the whole range of allowed parameters, while there is a significant bending at around $M = 500$ GeV in the dependence on M.

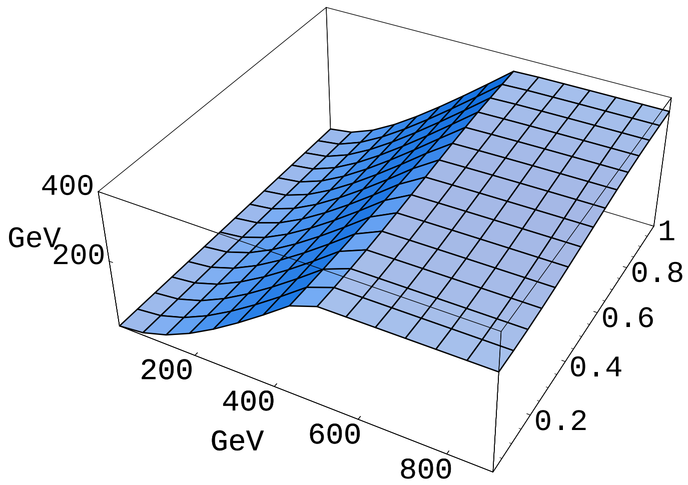

Figure 2: The mass of the LSP as function of M and δ. Plot of the lowest eigenvalues of the neutralino mass matrix for values $\tan(\beta) = 3$, $\mu = 500$, $m_2 = 400$, $m_1 = 300$, $m_S = 500$ as a function of M and δ.

If indeed $\chi^0_{\rm St}$ is the LSP then aside from the issue of a re-analysis of dark matter, the supersymmetric signals would be drastically modified. The usual missing energy signals where the lightest neutralino χ^0_1 is the LSP do not apply. Indeed if $\chi^0_{\rm St}$ lies lower than χ^0_1, then χ^0_1 will be unstable and will decay into $\chi^0_{\rm St}$ by a variety of decays channels such as

$$\chi^0_1 \quad\to\quad q_i\bar{q}_i\chi^0_{\rm St}\,, \quad l_i\bar{l}_i\chi^0_{\rm St}\,, \quad {\rm Z}\chi^0_{\rm St}\,. \tag{4.28}$$

We estimate the lifetime for this decay to lie in the range $10^{-19\pm2}$ sec. Thus χ^0_1 will decay in the detection chamber. In this case the detection signals will change drastically. Thus, for example, the decay of the chargino $\chi^-_1 \to l^- + \{\chi^0_1 + \nu_l\}$ will be changed into $\chi^-_1 \to l^-_1 l^-_2 l^+_2 + \{\chi^0_{\rm St} + \nu_l\}$. Similarly the decay of the slepton will lead also to a possible three lepton final state, i.e. $\tilde{l}^- \to l^- l^-_1 l^+_1 + \{\chi^0_{\rm St} + \nu_l\}$ while the well known decay of the off-shell W, i.e. ${\rm W}^* \to \chi^-_1\chi^0_2$, which in SUGRA models gives a trileptonic signal [19], in the present context can give rise to final states with three, five and seven leptons. Thus, we see that in this case there will be quite a significant change in the analysis of the phenomenology in search for supersymmetry. However, if the mass difference between χ^0_1 and $\chi^0_{\rm St}$ is not substantial, then the $q_i\bar{q}_i$ and $l_i\bar{l}_i$ produced in the decay of the χ^0_1 may be too soft to be detected. In this case there would be no substantial change in the SUSY signatures.

Also of interest is the status of dark matter in the Stueckelberg extension. As noted above there are now six neutral fermionic states compared with four for the case of the MSSM. The parameter space of the model is now also larger involving in addition to the MSSM parameters also the parameters of the Stueckelberg sector. It is known that in mSUGRA model over a significant part of the parameter space the lightest MSSM neutralino is also the LSP and thus a candidate for cold dark matter (CDM). This is also the case in the Stueckelberg extension. There exists a significant part of the parameters space where the lightest MSSM neutralino is the LSP. In this case the lightest MSSM neutralino will still be the cold dark matter candidate and essentially all of the analysis on dark matter in supergravity models will go through. However, there exists a part of the parameter space of the Stueckelberg extension, where the Stueckelberg fermion can become the LSP as discussed above. In this case, the analysis of dark matter will change drastically. A detailed analysis of the relic density is outside the scope of the present work and requires a separate analysis.

4.4 Coupling of gauge bosons to the hidden sector

While the couplings of the Z′ boson to the visible sector quarks and leptons are suppressed because of small mixing angles, this is not the case for the couplings of the Z′ boson to the hidden sector fields. Thus, for example, the couplings of Z′ to the hidden sector current J_X^μ is given by

$$\mathcal{L}_{\mathrm{Z}'\cdot\mathrm{hid}} = [\cos(\psi)\cos(\phi) - \sin(\theta)\sin(\phi)\sin(\psi)]\, \mathrm{Z}'_\mu g_X J_X^\mu \tag{4.29}$$

which we can rewrite into chiral components, using

$$g_X J_X^\mu = \sum_i \left[g_L^i \bar{f}_{\mathrm{hid},i}\gamma^\mu(1-\gamma_5) f_{\mathrm{hid},i} + g_R^i \bar{f}_{\mathrm{hid},i}\gamma^\mu(1+\gamma_5) f_{\mathrm{hid},i}\right]. \tag{4.30}$$

Using the above the decay width of Z′ into hidden sector fermions is given by

$$\Gamma(\mathrm{Z}' \to f_{\mathrm{hid}}\bar{f}_{\mathrm{hid}}) = \frac{M_{\mathrm{Z}'}}{6\pi}[\cos(\psi)\cos(\phi) - \sin(\theta)\sin(\phi)\sin(\psi)]^2 \sum_i ((g_L^i)^2 + (g_R^i)^2)\,. \tag{4.31}$$

To get an estimate of the Z′ decay width into the hidden sector matter, we set $(g_L^i)^2/4\pi = (g_R^i)^2/4\pi \sim 10^{-2}$ and $M_{\mathrm{Z}'} = 250\,\mathrm{GeV}$, which gives $\Gamma(\mathrm{Z}' \to f_{\mathrm{hid}}\bar{f}_{\mathrm{hid}}) \lesssim 3\,\mathrm{GeV}$. This is to be compared with the decay width of the Z′ into visible sector quarks which lies in the MeV range. Thus we see that the decay of the Z′ into hidden sector matter is much larger compared to the decay width of the Z′ into visible sector matter. This is to be expected due to the fact that Z′ is dominantly composed of C_μ which couples with normal strength to the hidden sector matter.

Another implication of the Stueckelberg extension is that it implies the photon couples with the hidden sector, if such a sector exists, with a small typically irrational charge. Thus one may write this coupling in the form

$$\mathcal{L}_{A^\gamma\cdot\mathrm{hid}} = e' A_\mu^\gamma J_X^\mu \tag{4.32}$$

where $e' = -g_X\cos(\theta)\sin(\phi)$. We note that similar mini-charges arise in models with kinetic energy mixings [20], where there are stringent limits on the size of these charges. These limits depend critically on the masses of the mini-charged particles [21, 22]. Although the mechanism by which the mini-charges arise in the Stueckelberg model discussed here is very different, one expects similar constraints on the charges of such particles. However, as discussed at the end of section 7, the size of the mini-charge with which the photon couples with the hidden sector is highly model-dependent. Of course, having a hidden sector is optional and one may eliminate it altogether by setting $J_X^\mu = 0$.

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4.5 Corrections to $g_\mu - 2$

Since the Z interactions are modified in the Stueckelberg extension, there is a modification of the Z exchange contribution to $g_\mu - 2$. Further, there is an additional contribution to $g_\mu - 2$ from the Z$'$ exchange. We now compute these corrections. For the Z exchange contribution we find

$$\Delta g_\mu^Z = \frac{m_\mu^2 G_F}{12\pi^2\sqrt{2}}[(3-4\cos^2\theta_W)^2\beta_v^2 - 5\beta_a^2]\,, \tag{4.33}$$

where

$$\begin{aligned}
\beta_v &= \frac{\sqrt{g_2^2+g_Y^2}}{-g_2^2+3g_Y^2}\left[\frac{-g_2^2+3\cos^2\phi g_Y^2}{\sqrt{g_2^2+\cos\phi g_Y^2}}\cos\psi - 3g_Y\sin\phi\sin\psi\right], \\
\beta_a &= \left[\frac{g_2^2+\cos^2\phi g_Y^2}{g_2^2+g_Y^2}\right]^{\frac{1}{2}}\left[\cos\psi - \frac{g_Y}{\sqrt{g_2^2+\cos^2\phi g_Y^2}}\sin\phi\sin\psi\right],
\end{aligned} \tag{4.34}$$

where $G_F = (g_2^2+g_Y^2)/(4\sqrt{2}M_Z^2)$. For the Z$'$ exchange contribution we find

$$\Delta g_\mu^{Z'} = \frac{m_\mu^2 G_F M_Z^2}{12\pi^2\sqrt{2}M_{Z'}^2}[(3-4\cos^2\theta_W)^2\gamma_v^2 - 5\gamma_a^2]\,, \tag{4.35}$$

where

$$\begin{aligned}
\gamma_v &= \frac{\sqrt{g_2^2+g_Y^2}}{-g_2^2+3g_Y^2}\left[\frac{-g_2^2+3\cos^2\phi g_Y^2}{\sqrt{g_2^2+\cos\phi g_Y^2}}\sin\psi + 3g_Y\sin\phi\cos\psi\right], \\
\gamma_a &= \left[\frac{g_2^2+\cos^2\phi g_Y^2}{g_2^2+g_Y^2}\right]^{\frac{1}{2}}\left[\sin\psi + \frac{g_Y}{\sqrt{g_2^2+\cos^2\phi g_Y^2}}\sin\phi\cos\psi\right].
\end{aligned} \tag{4.36}$$

The SM limit is $\phi = 0 = \psi$, or $\beta_v = 1 = \beta_a$, $\gamma_v = 0 = \gamma_a$, and eq. (4.33) gives the well known result of the Z exchange contribution in the SM. Numerically, the deviations from the SM are significantly smaller than the SM Z contribution and thus not discernible at the current level of hadronic error [23, 24] and experimental accuracy [25] in the determination of $g_\mu - 2$.

5. Stueckelberg at a linear collider

There is a general consensus that the high energy collider to be built after the Large Hadron Collider (LHC) should be a Linear Collider [26] and may most likely be an International Linear Collider (ILC) [27]. The design energies of such a machine could be $\sqrt{s} = 500\,\mathrm{GeV}$ (NLC500) with a luminosity of as much as $50fb^{-1}yr^{-1}$ or even larger. In addition to being an ideal machine for detailed studies of the properties of low lying supersymmetric particles such as light chargino and and light sfermions, a linear collider is also an ideal machine for testing some features of the type of extension of the SM and of MSSM discussed here.

5.1 Cross-sections including the Z′ pole

In the following we investigate such phenomena, the possibility of discovering the extra Z′ boson arising in the Stueckelberg extension. We begin by computing the scattering cross section of the process

$$e^+(p_1) + e^-(p_2) \to \mu^+(q_1) + \mu^-(q_2)\,. \tag{5.1}$$

This process can proceed via the direct channel exchange of the photon, of the Z and of the Z′ boson. Using the lagrangian for the Stueckelberg extension, an analysis for the spin averaged differential cross section gives[8]

$$\begin{aligned}
\frac{d\sigma}{d\Omega}(e^+e^- \to \mu^+\mu^-) = {} & \frac{\pi\alpha^2}{2s}(1+z^2) + \frac{\alpha}{2\sqrt{2}}\frac{G_F M_Z^2(s-M_Z^2)}{((s-M_Z^2)^2 + s^2\Gamma_Z^2 M_Z^{-2})} \times \\
& \times (v_e v_\mu(1+z^2) + 2a_e a_\mu z) + \frac{G_F^2 M_Z^4 s}{16\pi((s-M_Z^2)^2 + s^2\Gamma_Z^2 M_Z^{-2})} \times \\
& \times ((v_e^2 + a_e^2)(v_\mu^2 + a_\mu^2)(1+z^2) + 8 v_e a_e v_\mu a_\mu z) + \\
& + \frac{\alpha}{2\sqrt{2}}\frac{G_F M_Z^2(s-M_{Z'}^2)}{((s-M_{Z'}^2)^2 + s^2\Gamma_{Z'}^2 M_{Z'}^{-2})}(v_e' v_\mu'(1+z^2) + 2a_e' a_\mu' z) + \\
& + \frac{G_F^2 M_Z^4 s}{16\pi((s-M_{Z'}^2)^2 + s^2\Gamma_{Z'}^2 M_{Z'}^{-2})}((v_e'^2 + a_e'^2)(v_\mu'^2 + a_\mu'^2)(1+z^2) + \\
& \qquad\qquad + 8 v_e' a_e' v_\mu' a_\mu' z) + \\
& + \frac{G_F^2 M_Z^4 s(s-M_Z^2)(s-M_{Z'}^2)}{8\pi((s-M_Z^2)^2 + s^2\Gamma_Z^2 M_Z^{-2})((s-M_{Z'}^2)^2 + s^2\Gamma_{Z'}^2 M_{Z'}^{-2})} \\
& \times ((v_\mu v_\mu' + a_\mu a_\mu')(v_e v_e' + a_e a_e')(1+z^2) + \\
& \quad + 2(v_\mu a_\mu' + a_\mu v_\mu')(v_e a_e' + a_e v_e')z)\,,
\end{aligned} \tag{5.2}$$

where $z = \cos(\vartheta)$ with ϑ the scattering angle in the center of mass, and v_e, v_μ, etc are defined as follows

$$\begin{aligned}
v_e &= v_\mu = (\beta_L + \beta_R)\sin^2(\theta_W) - \frac{1}{2}\beta_L\,, \\
a_e &= a_\mu = (\beta_L - \beta_R)\sin^2(\theta_W) - \frac{1}{2}\beta_L\,, \\
v_e' &= v_\mu' = (\gamma_L + \gamma_R)\sin^2(\theta_W) - \frac{1}{2}\gamma_L\,, \\
a_e' &= a_\mu' = (\gamma_L - \gamma_R)\sin^2(\theta_W) - \frac{1}{2}\gamma_L\,,
\end{aligned} \tag{5.3}$$

where

$$\beta_R = \left[\frac{1+\tan^2(\theta_W)}{1+\tan^2(\theta_W)\cos^2(\phi)}\right]^{\frac{1}{2}} \cos^2(\phi)\cos(\psi) - \frac{\sqrt{1+\tan^2(\theta_W)}}{\tan(\theta_W)}\sin(\phi)\sin(\psi)\,,$$

[8]As is conventional we have used the Breit-Wigner parametrization of the amplitudes near the Z and Z′ poles in the form used in the fits of the LEP and the Tevatron data.

$$
\begin{aligned}
\beta_L &= \frac{\sqrt{1+\tan^2(\theta_W)}}{1-\tan^2(\theta_W)}\left[\frac{1-\tan^2(\theta_W)\cos^2(\phi)}{\sqrt{1+\tan^2(\theta_W)\cos^2(\phi)}}\cos(\psi)+\tan(\theta_W)\sin(\phi)\sin(\psi)\right],\\
\gamma_R &= \left[\frac{1+\tan^2(\theta_W)}{1+\tan^2(\theta_W)\cos^2(\phi)}\right]^{\frac{1}{2}}\cos^2(\phi)\sin(\psi)+\frac{\sqrt{1+\tan^2(\theta_W)}}{\tan(\theta_W)}\sin(\phi)\cos(\psi),\\
\gamma_L &= \frac{\sqrt{1+\tan^2(\theta_W)}}{1-\tan^2(\theta_W)}\left[\frac{1-\tan^2(\theta_W)\cos^2(\phi)}{\sqrt{1+\tan^2(\theta_W)\cos^2(\phi)}}\sin(\psi)-\tan(\theta_W)\sin(\phi)\cos(\psi)\right].
\end{aligned}
$$

Eq. (5.2) contains six different type of terms. These consist of three direct channel poles corresponding to the direct s channel exchange of the photon, the Z boson and the Z′ boson, and three interference terms which consist of the interference between the photon and the Z boson exchanges, the interference between the photon and the Z′ boson exchanges, and the interference between the Z boson and the Z′ boson exchanges. The entire effect of the Stueckelberg extension are contained in the parameters β_L, β_R, γ_L, and γ_R. Here β_L and β_R give the modification of the Z exchange interactions due to the Stueckelberg extension, and of course the Z′ interactions arise exclusively from the Stueckelberg extension. Thus the SM limit corresponds to $\beta_L = 1 = \beta_R$, and $\gamma_L = 0 = \gamma_R$. We note that the $\Gamma_{Z'}$ can also get contributions from the decay into the hidden sector.

Also of interest is the scattering cross section of the process

$$
e^+(p_1)+e^-(p_2)\to q(q_1)+\bar{q}(q_2)\,. \tag{5.4}
$$

Again this process can proceed via the direct channel exchange of the photon, of the Z and of the Z′ boson. Using the lagrangian for the Stueckelberg extension, an analysis for the spin averaged differential cross section gives

$$
\begin{aligned}
\frac{d\sigma}{d\Omega}(e^+e^-\to q\bar{q}) = &\ \frac{3\pi\alpha^2Q_q^2}{2s}(1+z^2) -\\
&-\frac{3\alpha Q_q}{2\sqrt{2}}\frac{G_FM_Z^2(s-M_Z^2)}{((s-M_Z^2)^2+s^2\Gamma_Z^2M_Z^{-2})}(v_ev_q(1+z^2)+2a_ea_qz)+\\
&+\frac{3G_F^2M_Z^4s}{16\pi((s-M_Z^2)^2+s^2\Gamma_Z^2M_Z^{-2})}((v_e^2+a_e^2)(v_q^2+a_q^2)(1+z^2)+\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad+8v_ea_ev_qa_qz)-\\
&-\frac{3\alpha Q_q}{2\sqrt{2}}\frac{G_FM_Z^2(s-M_{Z'}^2)}{((s-M_{Z'}^2)^2+s^2\Gamma_{Z'}^2M_{Z'}^{-2})}(v_e'v_q'(1+z^2)+2a_e'a_q'z)+\\
&+\frac{3G_F^2M_Z^4s}{16\pi((s-M_{Z'}^2)^2+s^2\Gamma_{Z'}^2M_{Z'}^{-2})}((v_e'^2+a_e'^2)(v_q'^2+a_q'^2)(1+z^2)+\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad+8v_e'a_e'v_q'a_q'z)+\\
&+\frac{3G_F^2M_Z^4s(s-M_Z^2)(s-M_{Z'}^2)}{8\pi((s-M_Z^2)^2+s^2\Gamma_Z^2M_Z^{-2})((s-M_{Z'}^2)^2+s^2\Gamma_{Z'}^2M_{Z'}^{-2})}\times\\
&\times((v_qv_q'+a_qa_q')(v_ev_e'+a_ea_e')(1+z^2)+\\
&\quad+2(v_qa_q'+a_qv_q')(v_ea_e'+a_ev_e')z)\,,
\end{aligned}
$$

where 3 is the color factor, $Q_u = \frac{2}{3}$, $Q_d = -\frac{1}{3}$. In the above v_q, a_q are defined as follows

$$
\begin{aligned}
v_q &= \frac{1}{2}[\delta_L \tau_3 - 2\sin^2(\theta_W) Q_q(\delta_L + \delta_R)]\,, \\
a_q &= \frac{1}{2}[\delta_L \tau_3 - 2\sin^2(\theta_W) Q_q(\delta_L - \delta_R)]\,,
\end{aligned}
\tag{5.5}
$$

where $\tau_3 = (1, -1)$ for $q = (u, d)$ and

$$
\begin{aligned}
\delta_R &= \delta_{\rm em} = \frac{\sqrt{1+\tan^2(\theta_W)\cos^2(\phi)}}{\sin^2(\theta_W)\sqrt{1+\tan^2(\theta_W)}}(\sin^2(\theta_W)\cos(\psi) - \sin(\theta)\tan(\phi)\sin(\psi))\,, \\
\delta_L &= \frac{\delta_3\tau_3 - 2\delta_{\rm em} Q_q \sin^2(\theta_W)}{\tau_3 - 2Q_q\sin^2(\theta_W)}\,, \\
\delta_3 &= \frac{\sqrt{1+\tan^2(\theta_W)\cos^2(\phi)}}{\sqrt{1+\tan^2(\theta_W)}}(\cos(\psi) - \sin(\theta)\tan(\phi)\sin(\psi))\,.
\end{aligned}
\tag{5.6}
$$

Similarly, in the above v'_q, a'_q are defined as follows

$$
\begin{aligned}
v'_q &= \frac{1}{2}[\epsilon_L \tau_3 - 2\sin^2(\theta_W) Q_q(\epsilon_L + \epsilon_R)]\,, \\
a'_q &= \frac{1}{2}[\epsilon_L \tau_3 - 2\sin^2(\theta_W) Q_q(\epsilon_L - \epsilon_R)]\,,
\end{aligned}
\tag{5.7}
$$

where

$$
\begin{aligned}
\epsilon_R &= \epsilon_{\rm em} = \frac{\sqrt{1+\tan^2(\theta_W)\cos^2(\phi)}}{\sin^2(\theta_W)\sqrt{1+\tan^2(\theta_W)}}(\sin^2(\theta)\sin(\psi) + \cos(\psi)\sin(\theta)\tan(\phi))\,, \\
\epsilon_L &= \frac{\epsilon_3\tau_3 - 2\epsilon_{em} Q_q \sin^2(\theta_W)}{\tau_3 - 2Q_q\sin^2(\theta_W)}\,, \\
\epsilon_3 &= \frac{\sqrt{1+\tan^2(\theta_W)\cos^2(\phi)}}{\sqrt{1+\tan^2(\theta_W)}}(\sin(\psi) + \cos(\psi)\sin(\theta)\tan(\phi))\,.
\end{aligned}
\tag{5.8}
$$

The SM limit is $\phi = 0 = \psi,\ \theta = \theta_W$, or $\delta_3 = 1, \delta_{\rm em} = 1, \epsilon_3 = 0, \epsilon_{\rm em} = 0$, and

$$
\begin{aligned}
v_q &= \frac{1}{2}(\tau_3 - 4Q_q\sin^2(\theta_W))\,, \\
a_q &= \frac{1}{2}\tau_3\,, \\
v'_q &= a'_q = 0\,,
\end{aligned}
\tag{5.9}
$$

which is correctly the SM result.

5.2 Forward-backward asymmetry near the Z′ pole

The forward-backward asymmetry is a useful tool in identifying the nature of the underlying interaction. One defines it as

$$
A_{fb} = \frac{\int_0^1 dz \frac{d\sigma}{dz} - \int_{-1}^0 dz \frac{d\sigma}{dz}}{\int_{-1}^1 dz \frac{d\sigma}{dz}}\,.
\tag{5.10}
$$

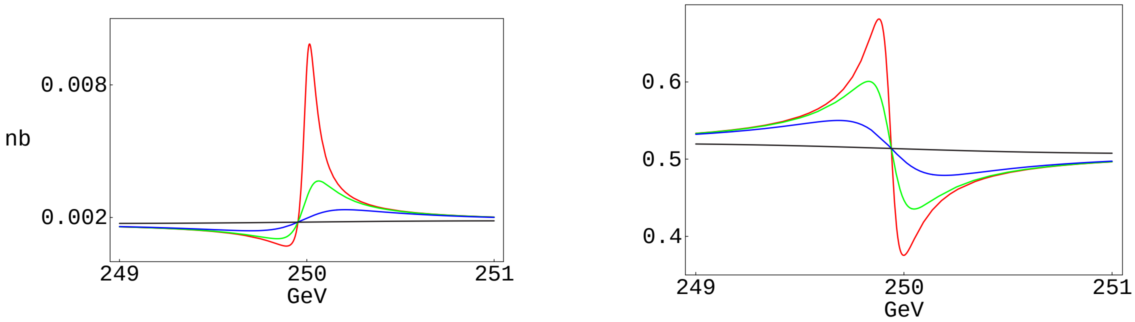

Figure 3: Plot of the total cross-section $\sigma(e^+e^- \to \mu^+\mu^-)$ (left) and the forward-backward asymmetry A_{fb} in $e^+e^- \to \mu^+\mu^-$ (right) in the vicinity of the Z′ resonance for $M_{Z'} = 250\,\mathrm{GeV}$, $\phi = 0.029$. The values of $\Gamma_{Z'}$ are 3 GeV (black line), 0.5 GeV (blue line), 0.2 GeV (green line), 0.08 GeV (red line).

Consider the case of $e^+e^- \to \mu^+\mu^-$ scattering. Here $\sigma_{\mu^+\mu^-} = \int_{-1}^{1} dz \frac{d\sigma}{dz}$ is given by

$$\begin{aligned}\sigma_{\mu^+\mu^-} &= \frac{4\pi\alpha^2}{3s} + \frac{2\sqrt{2}\alpha}{3}\frac{G_F M_Z^2(s-M_Z^2)v_e v_\mu}{((s-M_Z^2)^2+s^2\Gamma_Z^2 M_Z^{-2})} + \frac{G_F^2 M_Z^4 s(v_e^2+a_e^2)(v_\mu^2+a_\mu^2)}{6\pi((s-M_Z^2)^2+s^2\Gamma_Z^2 M_Z^{-2})} + \\ &+\frac{2\sqrt{2}\alpha}{3}\frac{G_F M_Z^2(s-M_{Z'}^2)v'_e v'_\mu}{((s-M_{Z'}^2)^2+s^2\Gamma_{Z'}^2 M_{Z'}^{-2})} + \frac{G_F^2 M_Z^4 s(v_e'^2+a_e'^2)(v_\mu'^2+a_\mu'^2)}{6\pi((s-M_{Z'}^2)^2+s^2\Gamma_{Z'}^2 M_{Z'}^{-2})} + \\ &+\frac{G_F^2 M_Z^4 s(s-M_Z^2)(s-M_{Z'}^2)(v_\mu v'_\mu + a_\mu a'_\mu)(v_e v'_e + a_e a'_e)}{3\pi((s-M_Z^2)^2+s^2\Gamma_Z^2 M_Z^{-2})((s-M_{Z'}^2)^2+s^2\Gamma_{Z'}^2 M_{Z'}^{-2})}\,. \end{aligned} \tag{5.11}$$

Using the above we can write the forward-backward asymmetry for this case so that

$$\begin{aligned}\sigma_{\mu^+\mu^-}A_{fb}^{\mu^+\mu^-} &= \frac{\alpha}{\sqrt{2}}\frac{G_F M_Z^2(s-M_Z^2)a_e a_\mu}{((s-M_Z^2)^2+s^2\Gamma_Z^2 M_Z^{-2})} + \frac{G_F^2 M_Z^4 s v_e a_e v_\mu a_\mu}{2\pi((s-M_Z^2)^2+s^2\Gamma_Z^2 M_Z^{-2})} + \\ &+\frac{\alpha}{\sqrt{2}}\frac{G_F M_Z^2(s-M_{Z'}^2)a'_e a'_\mu}{((s-M_{Z'}^2)^2+s^2\Gamma_{Z'}^2 M_{Z'}^{-2})} + \frac{G_F^2 M_Z^4 s v'_e a'_e v'_\mu a'_\mu}{2\pi((s-M_{Z'}^2)^2+s^2\Gamma_{Z'}^2 M_{Z'}^{-2})} + \\ &+\frac{G_F^2 M_Z^4 s(s-M_Z^2)(s-M_{Z'}^2)(v_\mu a'_\mu + a_\mu v'_\mu)(v_e a'_e + a_e v'_e)}{4\pi((s-M_Z^2)^2+s^2\Gamma_Z^2 M_Z^{-2})((s-M_{Z'}^2)^2+s^2\Gamma_{Z'}^2 M_{Z'}^{-2})}\,. \end{aligned} \tag{5.12}$$

We now start to discuss the numerical results for the total cross section at the Z′ pole, including or excluding the possibility of hidden sector matter fields it couples to. At the same time, we display the modifications of the forward-backward asymmetry near the pole.

In figure 3 we give a plot of the cross-section and A_{fb} for $e^+e^- \to \mu^+\mu^-$. First the largeness of the A_{fb} for the SM in this region comes from the γ–Z interference term which is large because of the the axial-vector coupling of the Z boson to fermions. When the Z′ contribution is included one finds a vary rapid variation in the vicinity of the Z′ pole, arising from two sources: the Z′ pole contribution to the asymmetry and the γ – Z′ contribution to the asymmetry. These contributions become large in the vicinity of the Z′ pole and compete in size with the SM contribution from γ–Z and Z. In figure 3 the largest peak corresponds to the case when there is no hidden sector. The width $\Gamma_{Z'}$ is determined for the decay of Z′ only into the fields of the visible sector, and approximating it by inclusion

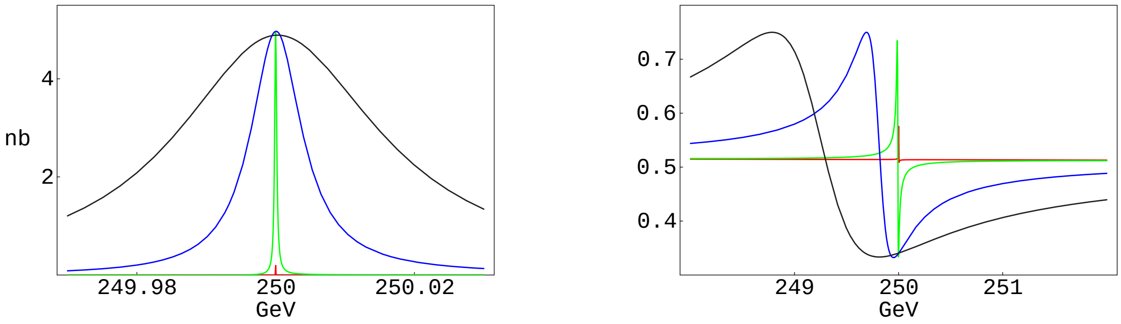

Figure 4: Plot of the total cross-section $\sigma(e^+e^- \to \mu^+\mu^-)$ (left) and the forward-backward asymmetry A_{fb} in $e^+e^- \to \mu^+\mu^-$ (right) in the vicinity of the Z′ resonance for $M_{Z'} = 250\,\mathrm{GeV}$. The values of δ are 0.1 (black line), 0.05 (blue line), 0.01 (green line), 0.001 (red line).

of only the quark and lepton final states excluding the top quark contribution. In this case we find that A_{fb} changes rapidly as we move across the Z′ pole. Thus, an accurate dedicated measurement of A_{fb} should give a signal for this type of resonance.

One may also include a hidden sector in the analysis. This is easily done by using eq. (4.31). As indicated in the analysis following eq. (4.31), Z′ couples with normal strength with the fields in the hidden sector and thus the decay width of the Z′ into the hidden sector fields need not be small, and indeed estimates show it to be of size $\mathcal{O}(\mathrm{GeV})$. In figure 3 we simulate the effects of the hidden sector by assuming a set of values for $\Gamma_{Z'}$ lying in the range 0.08 GeV to 3 GeV. One finds, as expected, that the effect of including the hidden sector is to make the peak in A_{fb} near the Z′ resonance less sharp. Thus the characteristics of A_{fb} in the vicinity of the Z′ resonance do indeed carry information regarding the presence or absence of a hidden sector. In figure 3 we also give a plot of $\sigma(e^+e^- \to \mu^+\mu^-)$ in the vicinity of the Z′ pole. One finds that the cross-section can be much larger relative to the SM result near the Z′ pole. In figure 4 an analysis of A_{fb} and of $\sigma(e^+e^- \to \mu^+\mu^-)$ for various values of δ but without a hidden sector is given. As expected one finds that the shape of the curves is a very sensitive function of δ with the resonance becoming broader as δ increases. One interesting feature of figure 4 is that the peak value of $\sigma(e^+e^- \to \mu^+\mu^-)$ is independent of δ. This is so because the peak value is essentially geometrical in nature and independent of δ as long as δ is small. This can be easily seen from eq. (6.1) by setting $E = M_{Z'}$. In this limit one finds that ratios $\Gamma(Z' \to e^+e^-(\mu^+\mu^-))/\Gamma(Z' \to all)$ appear. For small values of δ these ratios are independent of δ and take on the value

$$\sigma_{\mu^+\mu^-}(M_{Z'}) \simeq \frac{12\pi}{M_{Z'}^2} \times \begin{cases} (\frac{15}{103})^2 \\ (\frac{1}{8})^2 \end{cases} = \begin{cases} 4.8\,\mathrm{nb} & \text{for } M_{Z'} < 2m_t \\ 3.6\,\mathrm{nb} & \text{for } M_{Z'} > 2m_t \end{cases} . \tag{5.13}$$

One finds that the analysis of figure 4 is consistent with the analytic results on the peak value corresponding to the case $M_{Z'} < 2m_t$. We further note that the drop-off in $\sigma_{\mu^+\mu^-}$ away from the peak is very sharply dependent on δ. Further, A_{fb} deviates significantly from the SM prediction over a reasonable domain of the energy interval and provides another signature for the discovery of the Z′ resonance.

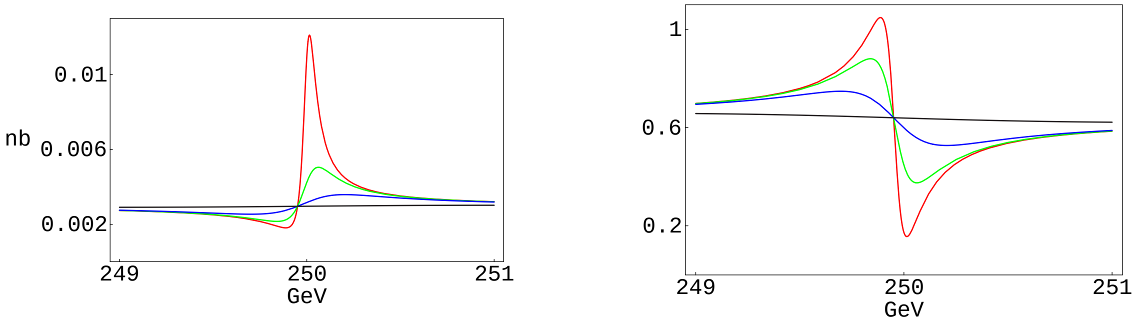

Figure 5: Plot of the total cross-section $\sigma(e^+e^- \to u\bar{u})$ (left) and the forward-backward asymmetry A_{fb} in $e^+e^- \to u\bar{u}$ (right) in the vicinity of the Z′ resonance for $M_{Z'} = 250\,\mathrm{GeV}$, $\phi = 0.029$. The values of $\Gamma_{Z'}$ are 3 GeV (black line), 0.5 GeV (blue line), 0.2 GeV (green line), 0.08 GeV (red line).

A similar analysis can be carried out for $e^+e^- \to q\bar{q}$. Here we have for the forward-backward asymmetry

$$\begin{aligned}\sigma_{q\bar{q}}A_{fb}^{q\bar{q}} = &-\frac{3\alpha Q_q}{\sqrt{2}}\frac{G_F M_Z^2(s-M_Z^2)a_e a_q}{((s-M_Z^2)^2+s^2\Gamma_Z^2 M_Z^{-2})}+\frac{3G_F^2 M_Z^4 s v_e a_e v_q a_q}{2\pi((s-M_Z^2)^2+s^2\Gamma_Z^2 M_Z^{-2})}-\\ &-\frac{3\alpha Q_q}{\sqrt{2}}\frac{G_F M_Z^2(s-M_{Z'}^2)a'_e a'_q}{((s-M_{Z'}^2)^2+s^2\Gamma_{Z'}^2 M_{Z'}^{-2})}+\frac{3G_F^2 M_Z^4 s v'_e a'_e v'_q a'_q}{2\pi((s-M_{Z'}^2)^2+s^2\Gamma_{Z'}^2 M_{Z'}^{-2})}+\\ &+\frac{3G_F^2 M_Z^4 s(s-M_Z^2)(s-M_{Z'}^2)(v_q a'_q+a_q v'_q)(v_e a'_e+a_e v'_e)}{4\pi((s-M_Z^2)^2+s^2\Gamma_Z^2 M_Z^{-2})((s-M_{Z'}^2)^2+s^2\Gamma_{Z'}^2 M_{Z'}^{-2})}\,, \end{aligned} \tag{5.14}$$

where

$$\begin{aligned}\sigma_{q\bar{q}} = &\frac{4\pi\alpha^2 Q_q^2}{s}-2\sqrt{2}\alpha Q_q\frac{G_F M_Z^2(s-M_Z^2)v_e v_q}{((s-M_Z^2)^2+s^2\Gamma_Z^2 M_Z^{-2})}+\frac{G_F^2 M_Z^4 s(v_e^2+a_e^2)(v_q^2+a_q^2)}{2\pi((s-M_Z^2)^2+s^2\Gamma_Z^2 M_Z^{-2})}-\\ &-2\sqrt{2}\alpha Q_q\frac{G_F M_Z^2(s-M_{Z'}^2)v'_e v'_q}{((s-M_{Z'}^2)^2+s^2\Gamma_{Z'}^2 M_{Z'}^{-2})}+\frac{G_F^2 M_Z^4 s(v_e^{\prime 2}+a_e^{\prime 2})(v_q^{\prime 2}+a_q^{\prime 2})}{2\pi((s-M_{Z'}^2)^2+s^2\Gamma_{Z'}^2 M_{Z'}^{-2})}+\\ &+\frac{G_F^2 M_Z^4 s(s-M_Z^2)(s-M_{Z'}^2)(v_q v'_q+a_q a'_q)(v_e v'_e+a_e a'_e)}{\pi((s-M_Z^2)^2+s^2\Gamma_Z^2 M_Z^{-2})((s-M_{Z'}^2)^2+s^2\Gamma_{Z'}^2 M_{Z'}^{-2})}\,. \end{aligned} \tag{5.15}$$

A numerical analysis of A_{fb} for the case when the final states are $u\bar{u}$ is given in figure 5, and again one finds that the characteristics of A_{fb} in the vicinity of the Z′ resonance are different for the cases: *i*) the SM, *ii*) the model with a Z′ resonance but without hidden sector matter, *iii*) models including decays of Z′ into the hidden sector.

An analysis of $\sigma(e^+e^- \to u\bar{u})$ is also given in figure 5. Here, again one finds that the cross section near the vicinity of the pole is significantly higher than the SM result and the deviation depends on the presence or absence of the possibility of decays into the hidden sector. Finally, we discuss the $e^+e^- \to d\bar{d}$. In figure 6 we give a plot of A_{fb} and one finds once again very significant deviations from the SM. As in previous cases the size of the deviation depends on the presence or absence of Z′ decays into the hidden sector.

The number of events for the various channels can be estimated by noting that at the projected design characteristics of 500 GeV collider one expects an integrated luminosity

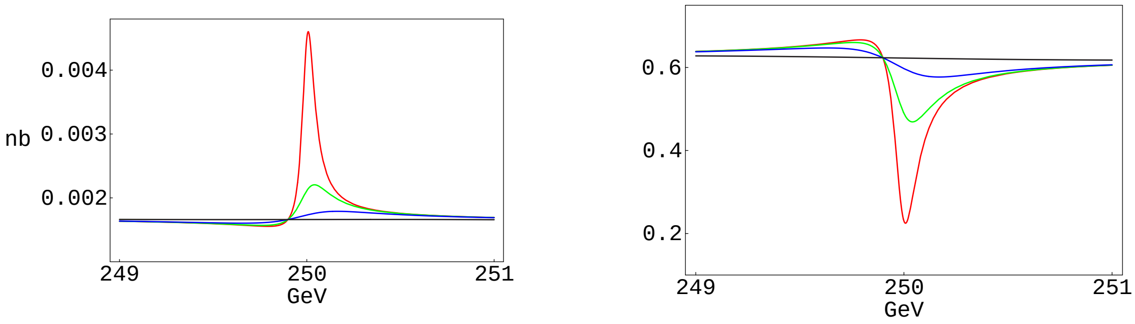

Figure 6: Plot of the total cross-section $\sigma(e^+e^- \to d\bar{d})$ (left) and the forward-backward asymmetry A_{fb} in $e^+e^- \to d\bar{d}$ (right) in the vicinity of the Z′ resonance for $M_{Z'} = 250\,\text{GeV}$, $\phi = 0.029$. The values of $\Gamma_{Z'}$ are 3 GeV (black line), 0.5 GeV (blue line), 0.2 GeV (green line), 0.08 GeV (red line).

of 500 $\text{fb}^{-1}\text{yr}^{-1}$, and the number of events using the cross sections of figures 3, 5, and 6 are clearly sizable. Finally, we note that an indication of the presence of a hidden sector to which the Z′ can decay will be provided by the visible width versus the total width of the Z′.

6. Detection of a sharp Z′ resonance

As mentioned already the Z′ is expected to be a sharp resonance, and determination of the $\Gamma(Z' \to e^+e^-)$ is a difficult problem as is well known from the analysis of the J/Ψ resonance [28]. A technique which was useful in the determination of the width of the J/Ψ should also be valid here, and this is the technique of integrating the cross section over the resonance [28]. Thus, for example, consider the cross-section for the process $e^+e^- \to f\bar{f}$ in the vicinity of the resonance. In this region one can write the cross-section so that[9]

$$\sigma(e^+e^- \to f\bar{f}) = \frac{3\pi}{M_{Z'}^2}\frac{\Gamma(Z' \to e^+e^-)\Gamma(Z' \to f\bar{f})}{((E - M_{Z'})^2 + \frac{1}{4}\Gamma^2(Z' \to \text{all}))}\,, \tag{6.1}$$

where $E = \sqrt{s}$. Integration over the resonance gives

$$\int dE\ \sigma(e^+e^- \to f\bar{f}) = \frac{6\pi^2}{M_{Z'}^2}\Gamma(Z' \to e^+e^-)\frac{\Gamma(Z' \to f\bar{f})}{\Gamma(Z' \to \text{all})}\,. \tag{6.2}$$

For a given final state we define

$$\mathcal{A}_{\text{fin}} = \int dE\ \sigma(e^+e^- \to \text{fin}) = \frac{6\pi^2}{M_{Z'}^2}\Gamma(Z' \to e^+e^-)\frac{\Gamma(Z' \to \text{fin})}{\Gamma(Z' \to \text{all})}\,, \tag{6.3}$$

and $\mathcal{A}_{\text{vis}}$ for the sum over all visible final states. Now for the case of Stueckelberg Z′ we have

$$\Gamma(Z' \to e^+e^-) \simeq \frac{\alpha_1}{8}M_{Z'}\tan^2(\phi)\,, \tag{6.4}$$

[9]Here we use the simplified form of the Breit-Wigner parametrization. Use of the more sophisticated form as in eq. (5.2) will give corrections to eq. (6.2) only of size $\mathcal{O}(\Gamma_{Z'}^2/M_{Z'}^2)$ which are very small.

where $\alpha_1 = g_1^2/4\pi$, and we have used the relation $g'_Y \simeq g_Y = \sqrt{\frac{3}{5}}g_1$. Further, under the assumption there is no hidden sector one has[10]

$$\frac{\Gamma(\mathrm{Z}' \to \mathrm{vis})}{\Gamma(\mathrm{Z}' \to \mathrm{all})} = \begin{cases} \frac{94}{103} & \text{for } M_{\mathrm{Z}'} < 2m_t \\ \frac{111}{120} & \text{for } M_{\mathrm{Z}'} > 2m_t \end{cases} . \tag{6.5}$$

Let us now focus on the final state $\mu^+\mu^-$. Using eqs. (6.2) and (6.4) we find

$$\mathcal{A}^{\mathrm{St}}_{\mu^+\mu^-} = \frac{45\pi^2}{412}\frac{\alpha_1 \tan^2(\phi)}{M_{\mathrm{Z}'}} . \tag{6.6}$$

For $M_{\mathrm{Z}'} = 250\,\mathrm{GeV}$, and $\delta = 0.02$ one finds $\mathcal{A}^{\mathrm{St}}_{\mu^+\mu^-} \simeq 1.1 \times 10^{-2}\,\mathrm{nb}$-GeV. Further, we note that the integral of eq. (6.6) falls as $1/M_{\mathrm{Z}'}$ as $M_{\mathrm{Z}'}$ gets large. Discovery of the Z′ depends on the signal versus the background. In this case the background is the SM contribution. Using the analysis of eq. (5.11) and excluding the Z′ contribution one finds that at $\sqrt{s} = M_{\mathrm{Z}'}$, $\sigma_{\mu^+\mu^-} = 1.8 \times 10^{-3}\,\mathrm{nb}$. If data is collected in bins of size Δ (in GeV) then the Standard Model $\mu^+\mu^-$ cross-section integrated over Δ around $M_{\mathrm{Z}'}$ gives $\mathcal{A}^{\mathrm{SM}}_{\mu^+\mu^-}(\Delta) = 1.8 \times 10^{-3}\Delta\,\mathrm{nb}$-GeV. Now for larger values of $M_{\mathrm{Z}'}$ the SM cross section falls as $1/M^2_{\mathrm{Z}'}$. Putting these factors together the ratio of the Stueckelberg contribution to the SM is given by

$$\frac{\mathcal{A}^{\mathrm{St}}_{\mu^+\mu^-}}{\mathcal{A}^{\mathrm{SM}}_{\mu^+\mu^-}(\Delta)} \simeq \frac{6}{\Delta(\mathrm{GeV})}\left[\frac{\delta}{0.02}\right]^2 \frac{M_{\mathrm{Z}'}(\mathrm{GeV})}{250} . \tag{6.7}$$

The above implies that for $M_{\mathrm{Z}'} = 250\,\mathrm{GeV}$, and $\delta = 0.02$, the Stueckelberg effects will give significant enhancement over the SM result with bin sizes ranging from $1 - 20\,\mathrm{GeV}$. Further, the signal to background ratio will increase as $M_{\mathrm{Z}'}$ increases. For example, for $M_{\mathrm{Z}'} = 1\,\mathrm{TeV}$, there will be a further enhancement of roughly a factor of 4. The above characteristics are encouraging. Of course, the detection of such an effect will depend on the design characteristics of the machine such as the beam spread, and the design luminosity.

The result of eq. (6.7) is also encouraging for the search for a Stueckelberg Z′ boson at the Tevatron and at the LHC. Here one would look for dilepton events in the final state via the Drell-Yan process and at the Tevatron it is the e^+e^- channel which would be the most efficient for detection.[11] The cross-sections for the processes $u\bar{u} \to l^+l^-$ and $d\bar{d} \to l^+l^-$ given here can be utilized for the computation of the Drell-Yan production of e^+e^- via the Stueckelberg Z′. Our analysis for the linear colliders hints that the detection of a sharp Z′ should also be possible at the hadron colliders. For example, at the Tevatron the energy resolution is given roughly by [29] $(15\%/\sqrt{E(\mathrm{GeV})} + 1\%)$ where E for our case would effectively be the di-muon invariant mass. Thus, for example, for $E = 250\,\mathrm{GeV}$ one has a resolution of about 5 GeV. This resolution should allow for a search for a resonance with characteristics of the type of eq. (6.7).

[10] In the computation of the ratio in eq. (6.5) we have included only quark and lepton final states. Inclusion of additional states, specifically the sparticle final states if they are allowed, will modify these ratios.

[11] We thank Darien Wood for pointing this out to us and also for bringing ref. [29] to our attention.

The radiative return technique might be a useful device to look for the Stueckelberg Z′ resonance. This is a useful procedure when the colliding beam energies have been fixed to a preassigned value and not continuously adjustable. In this case one uses initial state radiation (ISR) to reduce the effective center-of-mass energy [31]. Thus consider the process $e^+e^- \to \gamma +$ hadrons, where the γ is a hard photon which is emitted by one of the initial particles, and is responsible for reducing the center-of-mass energy. The method allows one to investigate the entire energy region below the highest energy down to the threshold. However, appropriate corrections must be made to account for the possibility that the photon may be emitted by the final state, i.e. one must take into account the final state radiation (FSR). One advantage of this technique is that the systematics of measurements remain unchanged in the scan as one changes the energy while in conventional energy scans systematics must be fixed at each step.

7. Stueckelberg extension with many extra U(1)

The Stueckelberg technique is, of course, extendable to more than one extra U(1) gauge symmetry. In orientifold string compactifications with D-branes the number of axions in the model is derived from the dimensional reduction of the ten-dimensional RR forms in the spectrum of the theory, and given by some topological quantity, the number of relevant homological cycles of the internal space. In principle it is an arbitrary number.[12] For example, in the so-called intersecting D-brane models on toroidal backgrounds, it was found that four such scalars participate in the generalized Green-Schwarz mechanism, and may thus also couple to the abelian gauge fields of the model in form of the Stueckelberg lagrangian. In general, one may write the extended lagrangian with N_V abelian gauge fields and N_S axions

$$\mathcal{L}_{\rm St} = -\frac{1}{4}\sum_{i=1}^{N_V}(C_{\mu\nu i}C_i^{\mu\nu} + g_iC_{\mu i}J_i^\mu) - \frac{1}{2}\sum_{j=1}^{N_S}\left(\partial_\mu\sigma_j + \sum_{i=1}^{N_V}M_{ij}C_{\mu i}\right)^2 . \tag{7.1}$$

We have now summarized the hyper charge gauge boson as one among the abelian gauge fields, say for $i = 1$ we let $B_\mu = C_{\mu 1}$. The generalized $\mathrm{U}(1)^{N_V}$ gauge invariance is given by

$$\delta_i C_{\mu i} = \partial_\mu\lambda_i\,, \qquad \delta_i\sigma_j = -M_{ij}\lambda_i\,. \tag{7.2}$$

In a very similar vein one can extend the supersymmetric minimal model by many axions and many abelian gauge bosons, as in

$$\mathcal{L}_{\rm St} = \int d^2\theta d^2\bar\theta\,\sum_{j=1}^{N_S}\left(S_j + \bar S_j + \sum_{i=1}^{N_V}M_{ij}C_i\right)^2 , \tag{7.3}$$

where S_j and C_i are the chiral and vector multiplets that include the axions and gauge fields. One can now easily see that the effect of each axion is to give mass to exactly one gauge boson, at least generically. The mass term induced after gauge fixing is a sum of

[12] This is actually similar for the heterotic string, which was recently demonstrated in [32].

squares, and each linear combination of masses M_{ij}, reading the $N_S \times N_V$ matrix as a set of vectors in the N_V-dimensional space of abelian gauge fields, defines one massive direction. In other words, the kernel of M_{ij} defines the set of the surviving massless abelian vectors. So, generically, all axions will be eaten by vectors, and only if there are more vectors than axions ($N_V > N_S$), or linear relations among their couplings will there be abelian gauge symmetries surviving.[13] If we further add the spontaneous electro-weak symmetry breaking through the Higgs mechanism, there is one more degree of freedom to be absorbed, and one more abelian vector receives a mass. Thus, if we intend to maintain an exactly massless photon in the very end, we have to make sure that the number of gauge bosons is at least two larger than the number of axions, which is exactly the situation of the minimal extensions in the StSM or StMSSM, which we introduced earlier.

In the supersymmetric extension the other degrees of freedom behave analogously. In the fermionic sector we gain a Stueckelberg chiral fermion for each S_j and a Stueckelberg gaugino for each C_i. These mix with the neutral fermions of the MSSM sector producing a neutralino mass matrix which is $4 + N_S + N_V$ dimensional. Model building with more than one Stueckelberg U(1) reduces the constraints on the mixing angles and thus provides a greater range of the parameter space for the discovery of new physics.

To illustrate this, let us briefly discuss the next simplest case, with two extra abelian factors, and two axions. The mass matrix for the neutral gauge fields $V_\mu^T = (C_{\mu 3}, C_{\mu 2}, B_\mu = C_{\mu 1}, A_\mu^3)$ looks then

$$\begin{bmatrix} M_{32}^2 + M_{31}^2 & M_{32}M_{22} + M_{31}M_{21} & M_{32}M_{12} + M_{31}M_{11} & 0 \\ M_{32}M_{22} + M_{31}M_{21} & M_{22}^2 + M_{21}^2 & M_{22}M_{12} + M_{21}M_{11} & 0 \\ M_{32}M_{12} + M_{31}M_{11} & M_{22}M_{12} + M_{21}M_{11} & M_{11}^2 + M_{12}^2 + \frac{1}{4}g_Y^2 v^2 & -\frac{1}{4}g_Y g_2 v^2 \\ 0 & 0 & -\frac{1}{4}g_Y g_2 v^2 & \frac{1}{4}g_2^2 v^2 \end{bmatrix}$$

where M_{ij} is the Stueckelberg coupling of $C_{\mu i}$ to the axion σ_j, $i = 1, 2, 3$ and $j = 1, 2$, and $C_{\mu 1} = B_\mu$. The matrix can be written as the sum of two contributions for the Stueckelberg terms and one for the Higgs effect, each one of which has only one non-vanishing eigenvalue, i.e. giving mass to one linear combination of gauge fields.

We can diagnonalize this matrix by an orthogonal transformation $\mathcal{O}^{[1]}$ and the eigenstates $E_\mu^{[1]} = \mathcal{O}^{[1]} V_\mu$ are arranged so that $E_\mu^{[1]} = (\mathrm{Z}''_\mu, \mathrm{Z}'_\mu, \mathrm{Z}_\mu, A_\mu^\gamma)^T$. The existence of two extra U(1) factors now, for instance, relaxes the constraints on photonic couplings to the hidden sector matter fields. As another application we demonstrate that the correction to the mass of the Z boson can now stay rather small even with comparatively large off-diagonal terms in the mass matrix. In figure 7 we have plotted the mass of the Z boson, the lightest non-vanishing eigenvalue of the mass matrix above as a function of M_{11} and M_{12}.

The left plot shows clearly that there is a range of parameters, where the effect of turning on the two off-diagonal elements partly cancels out, and M_{Z} falls off slower than along the axes. This happens roughly along the line $M_{11} = 0.6\, M_{12}$. In the right plot, the mass of Z is being plotted along this line, and for various overall mass scales (measured by

[13]This is an important constraint in the construction of string theoretic brane world models, where one has to impose extra constraints on the brane configurations to achieve this.

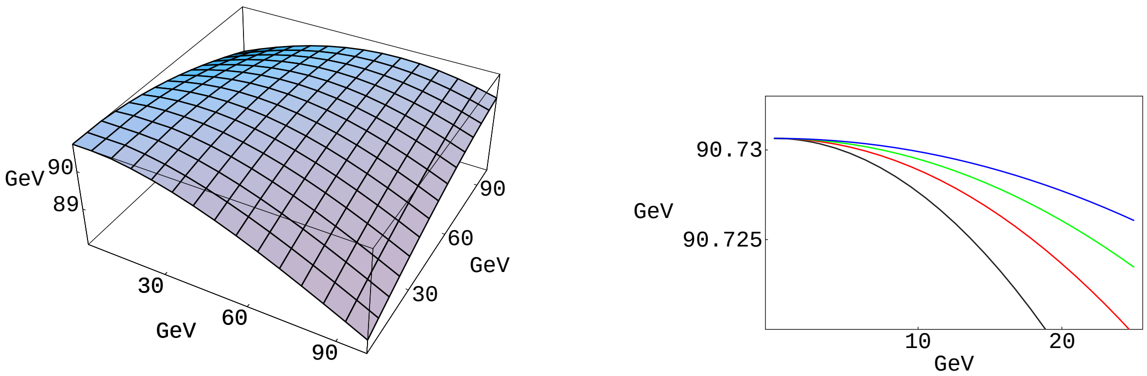

Figure 7: Plot of the mass of the Z, as a function of M_{11} and M_{12}, varying from 0 to 100 GeV in the left plot, and plotted along the variable $M_{11} = 0.6\,M_{12}$ varying from 0 to 25 GeV in the right one. The other mass parameters are chosen $M_{32} = 250\,\lambda\,\text{GeV}, M_{31} = 550\,\lambda\,\text{GeV}, M_{22} = 350\,\lambda\,\text{GeV}, M_{21} = 250\,\lambda\,\text{GeV}$. The value for λ is 1 in the left plot, and 1 (black line), 1.3 (red line), 1.6 (green line), and 2 (blue line) in the right plot.

λ) of the other parameters, differing by up to a factor of two. It is evident that up to values of 25 GeV the effect on the Z mass is still within some $10 - 30$ MeV. The mass eigenvalues of the matrix for $\lambda = 1$ are actually $\{719.4, 180.8, 90.7, 0\}$ in GeV, given the values used in figure 7. Together, this shows how Stueckelberg extensions with multiple U(1) factors have an even richer parameter space, which involves many more options to escape experimental bounds.

8. Conclusion

In this paper we have given a detailed analysis of the Stueckelberg extension of the electroweak sector of the SM and of the MSSM with an extra U(1) gauge group. This results in a new heavy gauge boson Z' whose couplings to leptons and quarks have vector and axial-vector couplings, which are different from those for the Z boson and of the Z' bosons in conventional $U(1)'$ extensions [15]. Additional new features arise for the Stueckelberg U(1) extension of MSSM, where the extension involves an abelian gauge superfield and a Stueckelberg chiral superfield consisting. The imaginary part of the complex scalar is absorbed in making the U(1) gauge vector massive, leaving a spin zero scalar. It is shown that this state is heavier than the Z'. Further, the neutral fermionic sector of the MSSM extension is also significantly extended. In addition to the four neutralino states of MSSM one has the Stueckelberg chiral fermion and the extra gaugino, which combine with the four MSSM neutral states to produce a 6×6 neutralino mass matrix. One interesting new possibility that arises here is the case where the LSP is mostly composed of the new fermions. In this case the lightest neutralino of the MSSM itself will be unstable leading to a possible new superweak candidate for dark matter. In the MSSM extension we also considered inclusion of the Fayet-Illiopoulos D-terms and discussed their implications.

A number of phenomenological implications were discussed in section 4. It was shown that the decay branching ratios of the Z′ into quarks and leptons are significantly different from the Z boson, which could provide a signature for the Stueckelberg origin of the Z′. We also discussed the Higgs sector of the extended MSSM model, where the mass matrix becomes a 3×3 matrix which mixes the residual spin zero field of the Stueckelberg chiral multiplet with the two CP-even neutral Higgs of MSSM. The mixings between the MSSM Higgs and the residual Stueckelberg spin state will produce a couplings of the latter with visible sector fermions and its main decay mode into visible fields is into the third generation quarks. We also discussed in section 4 the corrections to $g_\mu - 2$ and to sfermion masses.

In section 5 we gave an analysis of some of the signatures of the Z′ boson at a linear collider, such as the cross-sections $\sigma(e^+e^- \to \mu^+\mu^-)$, $\sigma(e^+e^- \to u\bar{u})$, and $\sigma(e^+e^- \to d\bar{d})$. In the vicinity of the resonance they differ significantly from the SM prediction. Further, the forward-backward asymmetry for the three cases discussed above deviates sharply from the SM, again providing an interesting signal. An interesting phenomenon is the effect of a hidden sector on the analysis. Thus, if a hidden sector with sufficiently light matter exists, so that the Z′ boson can decay into it, then the total width of the Z′ will be broadened. This has drastic effect on $\sigma(e^+e^- \to f\bar{f})$, and on the forward-backward asymmetry.

In section 6 we discussed the technique for the detection of a sharp resonance that is characteristic of the Stueckelberg extension. Finally, we have elaborated on the Stueckelberg extension of the electro-weak sector by an arbitrary number of extra U(1) factors. An interesting property of such models is the possibility that constraints on the parameters which mix the SM gauge bosons and the extra gauge bosons can be relaxed, allowing for the possibility of a richer phenomenology.

It should be interesting to carry out global fits to the electro-weak data and to explore further the testability of the Stueckelberg extension at colliders and in non-accelerator experiments.

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